Advanced Business Mathematics

SAMPLE TIME CONSTRAINED ASSESSMENT

Section A	**You must answer this question.**
Section B	**Answer any FOUR (4) questions from this section.** **Clearly cross out surplus answers.** **Failure to do this will result in only the first FOUR (4) answers being marked.**

Time: 4 hours

The maximum mark for this paper is 100.

Any reference material brought into the examination room must be handed to the invigilator before the start of the examination.

A formula sheet is provided at the end of the question paper.

Candidates are allowed to use a scientific calculator during this examination.

Graph paper will be provided by the centre.

You must show your workings.
Marks are awarded for these in all sections.

Section A

You must answer this question

Marks

Question 1

a) The data below shows the annual *per capita* (per person) consumption of three items over a two-year period.

	2019		2020	
	Price $	**Quantity (kg)**	**Price ($)**	**Quantity (kg)**
Item 1	8.26	15 520	9.10	16 100
Item 2	3.85	9520	3.95	9420
Item 3	12.58	7850	13.15	8150

Using **2019** as the base year calculate:

i) the *Laspeyres price index* for 2020. Give your answer to TWO (2) decimal places. **3**

ii) the Paasche price index for 2019. Give your answer to TWO (2) decimal places. **3**

b) For the equation: **3**

$$y = 5x^2\, ln(2x)$$

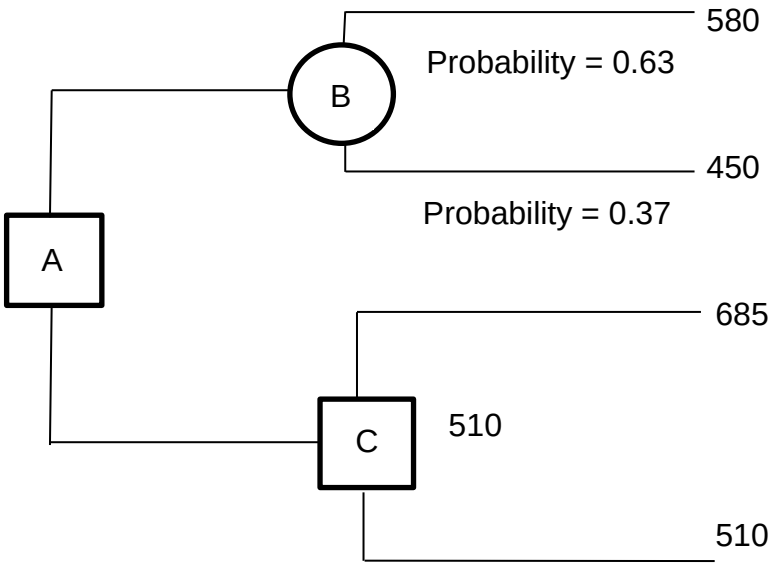
Differentiate y with respect to x.

c) Calculate the *Pearson Correlation Coefficient* for the set of sample observations given in the table below. **7**

							Σ
x	25	31	37	39	42	46	220
y	89	71	79	68	67	52	426

Marks

d) The following *decision tree* has been created by a management team.



i) Roll back the decision tree **and** find the values at nodes A **and** B. In finding the value for node A you will need to decide how the *optimisation rule* is being used. **4**

Total 20 Marks

Questions continue on the next page

Section B
Answer any FOUR (4) questions from this section

Marks

Question 2

A bakery produces two flavours of cake – A and B. The available demand is known for the following week.

Cake	Demand (no of cakes)
A	42
B	29

The company must also supply a minimum of 15 type A cakes for an existing contract with a regular customer.

The production of the cakes requires two manufacturing processes: the mixing phase and the baking phase. There are no input supply constraints.

The mixing phase has 210 hours available each month, and the baking phase has 350 hours available each month. The manufacturing times and profit for each type of table are given below:

	A	B
Mixing phase (time per cake)	15 minutes	12 minutes
Baking phase (time per cake)	50 minutes	35 minutes
Profit per cake	£3.50	£4.10

a) The company wishes to plan for the production of cakes for the following week with the intention of **maximising** profit. Formulate the problem as an objective function and associated set of inequalities. **You are not required to obtain a numerical solution to the problem.** **9**

b) The bakery decides to produce two more types of cake, C and D.

In order to plan for production next week with the intention of maximising profits, they formulate the problem as an objective function. They then use the Excel solver routine to solve the problem.

i) According to the 'Answer' report, the time available for baking is a *binding constraint*. Explain what this means. **2**

ii) According to the 'Sensitivity' report, the shadow price of the mixing time is zero. Explain why this is the case. **2**

iii) The 'Sensitivity' report shows that the baking time for the cakes has a shadow price of £5.60. Explain what this means. **2**

Marks

c) The bakery uses two ovens to bake the cakes, oven A and oven B.

Oven A bakes 45% of the cakes and oven B bakes 55% of the cakes.

The probability that oven A undercooks a cake is 0.02
The probability that oven B undercooks a cake is 0.03

i) A cake is chosen at random. Calculate the probability that it will be undercooked. **2**

ii) Given that a cake selected at random is undercooked, find the probability that the cake was baked in oven B (showing your workings). **3**

Total 20 Marks

Question 3

a) A factory, which produces rice, records the amount of rice in 10 bags as follows:

500g 498g 502g 507g 483g 495g 491g 493g 496g 501g

i) Calculate the *range* of the amount of rice in these 10 bags. **1**

ii) Calculate the *median* amount of rice in these 10 bags. **1**

iii) Calculate the *mean* amount of rice in these 10 bags. Show your workings. **1**

iv) Calculate the *sample variance* **and** *sample standard deviation* of the amount of rice in these 10 bags. **5**

b) The mean amount of rice in a bag is supposed to be 500g, but the factory suspects that a machine is underfilling the bags. Determine whether there is evidence to support this at the 5% significance level. You need to state the null **and** alternative hypothesis, the critical value of the test statistic **and** your conclusions. You should assume a normal distribution. Show your workings. **8**

c) Explain what is meant by a *Type 1 error* **and** how the chance of this type of error can be minimised. **2**

d) Explain what is meant by a *Type 2 error* **and** how the chance of this type of error can be minimised. **2**

Total 20 Marks

Marks

Question 4

a) A bookshop has recorded their book sales volume in hundreds (00s) over a FOUR (4) year period. The data is shown in the table below. **20**

i) Complete the analysis below using an *additive decomposition model* and *CMA 4*. With the aid of sketch graphs, comment upon the seasonality and trend.

Year	Quarter	Y Sales volume (00s)	T (CMA 4)
2017	Q1	78	
	Q2	69	
	Q3	75	76.625
	Q4	83	77.625
2018	Q1	81	78.75
	Q2	74	79.625
	Q3	79	80.375
	Q4	86	81.375
2019	Q1	84	82.5
	Q2	79	83.625
	Q3	83	
	Q4	91	
2020	Q1	89	
	Q2	82	
	Q3	85	
	Q4	96	

Total 20 Marks

Question 5

a) A company has developed a model for its demand curve:

$$P(q) = 58900 - 310q$$

Where $P(q)$ denotes the unit price in GBP (£) and q the quantity of items manufactured **and** sold.

i) Find an expression for *total revenue*, $R(q)$. Show your workings. **2**

ii) Differentiate the expression for the total revenue, $R(q)$, to find the gradient of $R(q)$. Show your workings. **2**

iii) Find the coordinates of the turning point of $R(q)$. Show your workings. **4**

iv) Sketch a graph of total revenue against output. You should label the axes **and** the turning point. State the maximum total revenue. **4**

Marks

b) 12 learner drivers are sitting their driving test. The probability of passing the driving test is 0.7. Calculate the probability that **at least** 10 of the learner drivers will pass the test. **4**

c) A clothing manufacturer forecasts next season's demand for a coat as: expected demand 1450 and a standard deviation 300. Assume demand is normally distributed. Calculate the probability that the actual demand will be between 1330 and 1585. **4**

Total 20 Marks

Question 6

a) Six candidates are interviewed for a job. The two interviewers rank the candidates as follows:

Candidate	A	B	C	D	E	F	Σ
Interviewer 1	5	1	2	4	6	3	
Interviewer 2	6	3	1	2	5	4	

i) Calculate the *Spearman Correlation Coefficient*. Show your workings. **5**

ii) Comment on the value of the *Spearman Correlation Coefficient.* **2**

b) A survey is carried out in a large city. 686 people out of a random sample of 980 said that they had shopped online in the past month. Determine a 99% confidence interval for the proportion of people in the city that have shopped online in the past month. **6**

c) A price index initially used 2015 as its base year. In 2019 the base year was updated.

YEAR	PRICE INDEX (2015 BASE)	PRICE INDEX (2019 BASE)
2015	100	94.1
2016	101.8	95.8
2017	102.7	96.6
2018	104.9	98.7
2019	106.3	100
2020		101.2

i) Link the two series to create a single series with 2019 as the base year. Give your answers to ONE (1) decimal place. **5**

ii) Use the single series to calculate the percentage change from 2017 to 2020 Give your answer to ONE (1) decimal place. **2**

Total 20 Marks

End of paper

Formula sheet
Management statistics

Population mean and standard deviation

$$\mu = \frac{\sum f_i x_i}{N} \qquad \mu = \sum p_i x_i$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{N}} \qquad \sigma = \sqrt{\sum p_i (x_i - \mu)^2}$$

Population Coefficient of Variation

$$CV = \frac{\sigma}{\mu}$$

Sample mean, standard deviation and sample variance

$$\bar{x} = \frac{\sum f_i x_i}{n} \qquad s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n-1}} \qquad \boldsymbol{s^2 = \frac{\Sigma (x - \bar{x})^2}{n-1}}$$

Sample skewness

$$\frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s} \right)^3$$

Sample Coefficient of Variation

$$CV = \frac{s}{\bar{x}}$$

Simple Index Number

$$R = \frac{p_n}{p_0} \times 100$$

Laspeyres and Paasche Price Index Numbers

$$LPI = 100 \times \frac{\sum q_o p_n}{\sum q_o p_o}$$

$$PPI = 100 \times \frac{\sum q_n p_n}{\sum q_n p_o}$$

Laspeyres and Paasche Quantity Index Numbers

$$LQI = 100 \times \frac{\sum q_n p_0}{\sum q_o p_o}$$

$$PQI = 100 \times \frac{\sum q_n p_n}{\sum q_0 p_n}$$

Probability

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Theorem of Bayes

$$P(E|A) = \frac{P(A|E)P(E)}{P(A)}$$

Theorem of Total Probability

$$P(A) = \sum_i P(A|E_i)P(E_i)$$

Binomial Distribution $B(n, p)$

$$P(X = x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{(n-x)}$$

$\mu = np$ $\quad\quad$ $\sigma = \sqrt{np(1-p)}$

Poisson Distribution $Po(\lambda)$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$\mu = \lambda$ $\quad\quad$ $\sigma = \sqrt{\lambda}$

Exponential Distribution

$f(t) = \lambda e^{-\lambda t},\ t \geq 0$

$P(T < t) = 1 - e^{-\lambda t}$

$\mu = \frac{1}{\lambda}$ $\quad\quad$ $\sigma = \frac{1}{\lambda}$

Standard Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$

$$f(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$$

Hypothesis Testing

Distribution of sample means

$\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Large sample confidence interval of the mean $(n \geq 30)$

$(\mu^-, \mu^+) = \left(\bar{x} - z_\gamma \frac{s}{\sqrt{n}}, \bar{x} + z_\gamma \frac{s}{\sqrt{n}}\right)$

Large sample confidence interval of proportion $(np \text{ and } n(1 - p) \geq 5)$

$(p^-, p^+) = \left(\bar{p} - z_\gamma \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \bar{p} + z_\gamma \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right)$

Exact confidence interval (underlying population has normal distribution)

$(\mu^-, \mu^+) = \left(\bar{x} - t_\gamma \frac{s}{\sqrt{n}}, \bar{x} + t_\gamma \frac{s}{\sqrt{n}}\right)$

Approximate large sample test of the mean

$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

Under the null hypothesis $Z \sim N(0, 1)$, approximately.

Student's one sample t-test of the mean.

$T = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

Under the null hypothesis $T \sim t(n - 1)$

Independent two sample t-test

$T = \frac{\overline{x_1} - \overline{x_2}}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ $S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Under the null hypothesis $T \sim t(n_1 + n_2 - 2)$

Fitting Data

χ^2 Goodness of fit test ($E_i \geq 5$ for all i)

$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

Under the null hypothesis $\chi^2 \sim \chi^2(k - m - 1)$

k is number of categories, m is number of model parameters estimated from data

χ^2 Test of Association ($E_i \geq 5$ for all i)

$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

Under the null hypothesis $\chi^2 \sim \chi^2\big((r-1)(c-1)\big)$

r is number of rows, c is number of columns

Simple Linear Regression

$\hat{y} = \mathrm{m}x_\mathrm{i} + \mathrm{c}$ is the least SSE straight line where;

$\mathrm{m} = \frac{\sum(x_\mathrm{i} - \bar{x})(y_\mathrm{i} - \bar{y})}{\sum(x_i - \bar{x})^2}$ $\quad$ $\mathrm{m} = \frac{\mathrm{n}\sum x_\mathrm{i}y_\mathrm{i} - \sum x_\mathrm{i}\sum y_\mathrm{i}}{\mathrm{n}\sum x_\mathrm{i}^2 - (\sum x_\mathrm{i})^2}$

$\mathrm{c} = \bar{y} - \mathrm{m}\bar{x}$

The Coefficient of Determination

$$R^2 = r^2 = \frac{\sum(\hat{y} - \bar{y})^2}{\sum(y - \bar{y})^2}$$

The Pearson Correlation Function

$$\mathrm{R} = \mathrm{r} = \frac{\mathrm{n}\sum x_\mathrm{i}y_\mathrm{i} - \sum x_\mathrm{i}\sum y_\mathrm{i}}{\sqrt{(\mathrm{n}\sum x_\mathrm{i}^2 - (\sum x_\mathrm{i})^2)(\mathrm{n}\sum y_\mathrm{i}^2 - (\sum y_\mathrm{i})^2)}}$$

Spearman's Rank Correlation (with no ties)

$$r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

Differentiation

Definition

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Standard Derivatives

$\boldsymbol{y}$	$\boldsymbol{\frac{dy}{dx}}$
$y = ax^n$	$\frac{dy}{dx} = nax^{\mathrm{n}-1}$
$y = \mathrm{e}^{ax}$	$\frac{dy}{dx} = a\mathrm{e}^{ax}$
$y = ln(ax)$ $= log_{\mathrm{e}}(x)$	$\frac{dy}{dx} = \frac{1}{x}$

Rules of Differentiation

$\frac{d}{dx}\left(af(x) + bg(x)\right) = a\frac{df}{dx} + b\frac{dg}{dx}$

$\frac{d}{dx}\left(f(x)g(x)\right) = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$

$\frac{d}{dx}\left(f\left(g(x)\right)\right) = \frac{df}{dg}\frac{dg}{dx}$

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{\left(g(x)\right)^2}$

Elasticities of Demand

Own price	Cross price	Income
$E_p = \frac{\partial Q_1}{\partial p_1}\frac{p_1}{Q_1}$	$E_{12} = \frac{\partial Q_1}{\partial p_2}\frac{p_2}{Q_1}$	$E_I = \frac{\partial Q_1}{\partial I}\frac{I}{Q_1}$

The Total Differential

$y = y(x_1 + x_2 + x_3 + \ldots)$

$dy = \frac{\partial y}{\partial x_1}dx_1 + \frac{\partial y}{\partial x_2}dx_2 + \frac{\partial y}{\partial x_3}dx_3 + \ldots$

$\Delta y \approx \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2 + \frac{\partial y}{\partial x_3}\Delta x_3 + \ldots$

Time series

The additive decomposition model

$Y_n = T_n + S_n + I_n$

The multiplicative decomposition model

$Y_n = T_n \times S_n \times I_n$

Three Point Moving Average

$T_n = \frac{1}{3}(Y_{n-1} + Y_n + Y_{n+1})$

Four Point Centred Moving Average

$T_n = \frac{1}{8}(Y_{n-2} + 2Y_{n-1} + 2Y_n + 2Y_{n+1} + Y_{n+2})$

Simple Exponential Smoothing

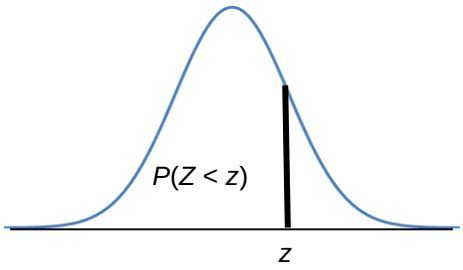
$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$

Errors

$$MSE = \frac{1}{N}\sum_{j=1}^{N}\left(Y_j - F_j\right)^2$$

$$MAE = \frac{1}{N}\sum_{j=1}^{N}\left|Y_j - F_j\right|$$

Probabilities under the Normal Distribution Curve

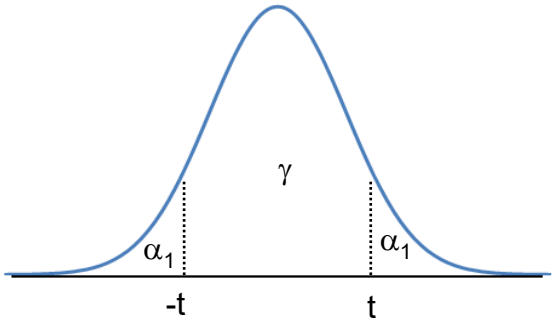


z	0	1	2	3	4	5	6	7	8	9
-3.50	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.00	**0.5000**	**0.4960**	**0.4920**	**0.4880**	**0.4840**	**0.4801**	**0.4761**	**0.4721**	**0.4681**	**0.4641**

z	0	1	2	3	4	5	6	7	8	9
0.00	**0.5000**	**0.5040**	**0.5080**	**0.5120**	**0.5160**	**0.5199**	**0.5239**	**0.5279**	**0.5319**	**0.5359**
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Percentage Points of the Student t Distribution

	α_1	**5.00%**	2.50%	**1.00%**	**0.50%**
	α_2	10.00%	**5.00%**	2.00%	**1.00%**
	γ	90.00%	**95.00%**	98.00%	**99.00%**
df	1	6.3138	12.7062	31.8205	63.6567
	2	2.9200	4.3027	6.9646	9.9248
	3	2.3534	3.1824	4.5407	5.8409
	4	2.1318	2.7764	3.7469	4.6041
	5	2.0150	2.5706	3.3649	4.0321
	6	1.9432	2.4469	3.1427	3.7074
	7	1.8946	2.3646	2.9980	3.4995
	8	1.8595	2.3060	2.8965	3.3554
	9	1.8331	2.2622	2.8214	3.2498
	10	1.8125	2.2281	2.7638	3.1693
	11	1.7959	2.2010	2.7181	3.1058
	12	1.7823	2.1788	2.6810	3.0545
	13	1.7709	2.1604	2.6503	3.0123
	14	1.7613	2.1448	2.6245	2.9768
	15	1.7531	2.1314	2.6025	2.9467
	16	1.7459	2.1199	2.5835	2.9208
	17	1.7396	2.1098	2.5669	2.8982
	18	1.7341	2.1009	2.5524	2.8784
	19	1.7291	2.0930	2.5395	2.8609
	20	1.7247	2.0860	2.5280	2.8453
	21	1.7207	2.0796	2.5176	2.8314
	22	1.7171	2.0739	2.5083	2.8188
	23	1.7139	2.0687	2.4999	2.8073
	24	1.7109	2.0639	2.4922	2.7969
	25	1.7081	2.0595	2.4851	2.7874
	26	1.7056	2.0555	2.4786	2.7787
	27	1.7033	2.0518	2.4727	2.7707
	28	1.7011	2.0484	2.4671	2.7633
	29	1.6991	2.0452	2.4620	2.7564
	30	1.6973	2.0423	2.4573	2.7500
	31	1.6955	2.0395	2.4528	2.7440
	32	1.6939	2.0369	2.4487	2.7385
	33	1.6924	2.0345	2.4448	2.7333
	34	1.6909	2.0322	2.4411	2.7284
	35	1.6896	2.0301	2.4377	2.7238
	36	1.6883	2.0281	2.4345	2.7195
	37	1.6871	2.0262	2.4314	2.7154
	38	1.6860	2.0244	2.4286	2.7116
	39	1.6849	2.0227	2.4258	2.7079
	40	1.6839	2.0211	2.4233	2.7045
	∞	**1.6449**	**1.9600**	**2.3263**	**2.5758**



Critical Values for the χ^2 Distribution

df	0.05 α_R **5.00%**	0.01 **1.00%**
1	3.841	6.635
2	5.991	9.210
3	7.815	11.345
4	9.488	13.277
5	11.070	15.086
6	12.592	16.812
7	14.067	18.475
8	15.507	20.090
9	16.919	21.666
10	18.307	23.209
11	19.675	24.725
12	21.026	26.217
13	22.362	27.688
14	23.685	29.141
15	24.996	30.578
16	26.296	32.000
17	27.587	33.409
18	28.869	34.805
19	30.144	36.191
20	31.410	37.566
21	32.671	38.932
22	33.924	40.289
23	35.172	41.638
24	36.415	42.980
25	37.652	44.314
26	38.885	45.642
27	40.113	46.963
28	41.337	48.278
29	42.557	49.588
30	43.773	50.892