



Business Mathematics

SAMPLE TIME CONSTRAINED ASSESSMENT MARKING SCHEME

This marking scheme has been prepared as a **guide only** to markers. This is not a set of model answers, or the exclusive answers to the questions, and there will frequently be alternative responses which will provide a valid answer. Markers are advised that, unless a question specifies that an answer be provided in a particular form, then an answer that is correct (factually or in practical terms) **must** be given the available marks.

If there is doubt as to the correctness of an answer, the relevant NCC Education materials should be the first authority.

Throughout the marking, please credit any valid alternative point.

Where markers award half marks in any part of a question, they should ensure that the total mark recorded for the question is rounded up to a whole mark.

1. Solution of quadratic equations

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Measures of location

Population mean

Ungrouped data:

$$\mu = \frac{\sum x}{N}$$

Ungrouped frequency table:

$$\mu = \frac{\sum fx}{N} = \frac{\sum fx}{\sum f}$$

Grouped frequency table: $\mu = \frac{\sum fm}{N} = \frac{\sum fm}{\sum f}$

Sample mean

Ungrouped data: $\sum_{x} x$

$$\bar{x} = \frac{2\pi}{n}$$

Ungrouped frequency table: $\bar{x} = \frac{\sum fx}{n} = \frac{\sum fx}{\sum f}$

Grouped frequency table: $\bar{x} = \frac{\sum fm}{n} = \frac{\sum fm}{\sum f}$

3. Measures of dispersion

Population variance

Ungrouped data:

$$\sigma^2 = \frac{\sum (x - \mu)^2}{N}$$

Population standard deviation

Ungrouped data:

$$\sigma = \sqrt{\frac{1}{N} \left[\sum (x - \mu)^2 \right]} = \sqrt{\frac{1}{N} \left[\sum x^2 - \frac{(\sum x)^2}{N} \right]}$$

Ungrouped frequency table:

$$\sigma = \sqrt{\frac{1}{N} \left[\sum f(x - \mu)^2 \right]} = \sqrt{\frac{1}{\sum f} \left[\sum fx^2 - \frac{(\sum fx)^2}{\sum f} \right]}$$

Grouped frequency table:

$$\sigma = \sqrt{\frac{1}{N} \left[\sum f(m-\mu)^2 \right]} = \sqrt{\frac{1}{(\sum f)} \left[\sum fm^2 - \frac{(\sum fm)^2}{\sum f} \right]}$$

Sample variance

Ungrouped data: $s^{2} = \frac{\sum (x - \bar{x})^{2}}{n - 1}$

Sample standard deviation

Ungrouped data:

$$s = \sqrt{\frac{1}{n-1} \left[\sum (x - \bar{x})^2 \right]} = \sqrt{\frac{1}{n-1} \left[\sum x^2 - \frac{(\sum x)^2}{n} \right]}$$

Ungrouped frequency table:

$$s = \sqrt{\frac{1}{n-1} \left[\sum f(x-\bar{x})^2 \right]} = \sqrt{\frac{1}{(\sum f) - 1} \left[\sum fx^2 - \frac{(\sum fx)^2}{(\sum f)} \right]}$$

Grouped frequency table:

$$s = \sqrt{\frac{1}{n-1} \left[\sum f(m-\bar{x})^2 \right]} = \sqrt{\frac{1}{(\sum f) - 1} \left[\sum fm^2 - \frac{(\sum fm)^2}{\sum f} \right]}$$

4. Exponential forecasting

 $F_{t+1} = F_t + \alpha (x_2 - F_t)$

Business Mathematics

Section A	
Answer ALL questions from this section	
	Marks
Question 1	
Simplify the ratio 12:9	1
Mark scheme	
4:3 (1 mark)	
Question 2	
Calculate: $60 - 20 \times 2$	1
Mark scheme	
20 (1 mark)	
Question 3	
Round 4025 to ONE (1) significant figure.	1
Mark scheme	
4000 (1 mark)	
Question 4	
Express 54203 in standard form.	1
Mark Scheme	
5.4203 \times 10 ⁴ (1 mark)	
Question 5	
Write TWO (2) days as a fraction of a week.	1
Mark scheme	

 $\frac{2}{7}$ (1 mark)

Question 6	
Calculate 2% of \$430	1
Mark scheme	
\$8.60 (1 mark)	
Question 7	
State the gradient of the line $10x - 9$	1
Mark scheme	
10 (1 mark)	
Question 8	
State the <i>y</i> -intercept of the line $y = 3x + \frac{1}{2}$	1
Mark scheme	
$\frac{1}{2}$ (1 mark)	
Question 9	
Simplify: $m^5 \times m^3$	1
Mark scheme	
m ⁸ (1 mark)	
Question 10	
Write $\frac{17}{3}$ as a mixed number.	1
Mark scheme	

$5\frac{2}{3}$ (1 mark)

Question 11

Calculate the THREE (3) 4-point moving averages for the following data:

101 152 147 136 104 124

Mark scheme

 $(101+152+147+136) \div 4 = 134$ (1 mark) $(152+147+136+104) \div 4 = 134.75$ (1 mark) $(147+136+104+124) \div 4 = 127.75$ (1 mark)

Question 12

Find: $\frac{4}{5} \times \frac{3}{4} + \frac{2}{3} \div \frac{1}{5}$

Show your workings fully.

Mark scheme

 $\frac{\frac{4}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5} (1 \text{ mark, simplified or not})$ $\frac{\frac{2}{3} \div \frac{1}{5} = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3} (1 \text{ mark})$ $\frac{3}{5} + \frac{10}{3} = \frac{9+50}{15} = \frac{59}{15} = 3\frac{14}{15} (1 \text{ mark})$

(Total 3 marks)

Question 13

Make *b* the subject of the following equation:

 $\frac{\sqrt{ab}}{d} = e$

Mark scheme

Multiply by d: $\sqrt{ab} = de$ Square both sides:

$$ab = (de)^2$$

Divide by a:

 $b = \frac{(de)^2}{2}$

- 1 mark for partially correct method, 2 marks for wholly correct method.
- Correct answer (1 mark)

(Total 3 marks)

3

3

Question 14

A savings account offers interest at a rate of 2.3% compounded annually. If Manisha opens an account with \pounds 240 and leaves it for EIGHT (8) years, calculate how much will be in the account at the end of EIGHT (8) years. Give your answer to the nearest pound (\pounds).

Mark scheme

The multiplier for an increase of 2.3% is 1.023 (1 mark for use of correct multiplier). So, £240 \times 1.023⁸ = £288 the nearest pound) (1 mark for method, 1 mark for correct answer, rounded correctly)

(Total 3 marks)

Question 15

Solve these *simultaneous equations* algebraically:

5x - 3y = 27

11x + 9y = -3

Mark scheme

Multiply the first equation by 3: 15x - 9y = 81Add the second equation:

15x - 9y = 8111x + 9y = -3

26x = 78

So, x = 3 (1 mark for working, 1 mark for value of x. Please note workings may vary between candidates but credit should be given for any valid workings.)

Substitute value of x into 5x - 3y = 27

5(3) - 3y = 27

-3y = 12

So y = -4 (1 mark)

(Total 3 marks)

Question 16

Solve the following equation by *completing the square*:

 $x^2 + 6x = -7$

You may leave your answer in surd form or round your answer to TWO (2) decimal places.

Mark Scheme

 $Add (3)^2 = 9$

Rewrite as: $x^2 + 6x + 9 = 2$

So, $(x+3)^2 = 2$

Take the square root of each side: $x + 3 = \pm \sqrt{2}$

So,

$$x = \pm \sqrt{2} - 3$$

So $x = -1.59$ (to 2. d.p.) or $x = -4.41$ (to 2 d.p.)

Award 1 mark for partially correct workings, award 2 marks for wholly correct workings. Correct answer – accept either surd form or rounded answer. (1 mark)

(Total 3 marks)

Question 17

200 people were asked to state their preferred brand of toothpaste. The results are below:

3

	Bright Smile	Fresh 'n' Minty	Smiths	Natural Smile
Frequency	98	62	12	28

A pie chart of the data is drawn to illustrate the results. Calculate the angles of the sectors for 'Bright Smile', 'Fresh 'n' Minty' and 'Natural Smile'.

You are not required to construct a pie chart.

Mark Scheme

Bright Smile: $\frac{98}{200} \times 360^{\circ} = 176.4^{\circ}$ (1 mark) Fresh 'n' Minty: $\frac{62}{200} \times 360^{\circ} = 111.6^{\circ}$ (1 mark) Natural Smile: $\frac{28}{200} \times 360^{\circ} = 50.4^{\circ}$ (1 mark)

Question 18

A shop sells phone cases. It has 120 phone cases in stock. 3/4 of the phone cases **3** are large and the rest are small. 80 of the phone cases are patterned and the rest are plain. If ONE (1) phone case is selected at random, calculate the probability that it will be a small, plain phone case. Assume that size is independent of whether phone case is patterned or plain.

Mark scheme

 $P(small) = \frac{1}{4} (1 mark)$ $P(plain) = \frac{40}{120} = \frac{1}{3} (1 mark)$ $P(small, plain) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Correct answer (1 mark)

(Total 3 marks)

Question 19

Calculate the mean, median and mode of the following data set:

3

30 27 25 32 27 25 30 31 30 26

Mark scheme

Mean: $\frac{283}{10} = 28.3$ (1 mark)

Median value: 28.5(1 mark)

Mode: 30 (1 mark)

Question 20

The number of customers booking holidays online increased by 2% in 2018 and 5% **3** in 2019. Calculate the overall *percentage change* in the number of customers booking holidays online.

Mark scheme

In 2018, the increase was 2%, so the multiplier is 1.02. In 2019, the increase was 5%, so the multiplier is 1.05

1.02 x 1.05 = 1.071

So, the overall percentage change was 7.1%

1 mark for partially correct method, 1 mark for wholly correct method.

Correct answer (1 mark)

(Total 3 marks)

Total 40 Marks

Section B Answer any THREE (3) questions from this section

Question 21

- a) An electrician charges $\pounds C$ for a job depending on h (amount of time in hours) according to the relationship: C = 45h + 50
 - i) Complete the following table for the graph C = 45h + 50

3

4

Marks

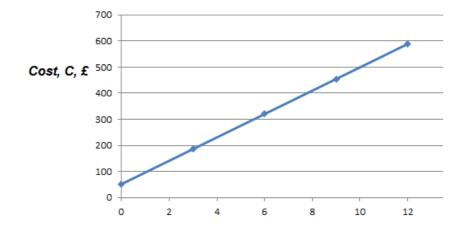
h	0	3	6	9	12
С	50			455	

Mark scheme

h	0	3	6	9	12
С	50	185 (1 mark)	320 (1 mark)	455	590 (1 mark)

ii) Plot the graph of C = 45h + 50 for $0 \le h \le 12$

Mark scheme



Time, h, hours

- Axis drawn and correctly labelled (1 mark)
- Award 1 mark for two data points correctly plotted. Award 2 marks for all 5 data points correctly plotted.
- Points joined with a straight line (1 mark)
- iii) Use the graph to find out how much it would cost for a job that takes FOUR (4) hours.

Mark scheme

£230 (1 mark)

2

iv) Use the graph to find out how many hours a job costing £297.50 takes.

Mark scheme

5 1/2 hours (1 mark)

v) Explain why a job cannot cost less than £50.

Mark scheme

The y-intercept is 50, i.e. £50, so the cost cannot be lower (1 mark). This is the fixed starting price (or call out fee) (1 mark).

1

b) Another electrician charges a £60 call out fee plus £35 per hour. Write this information as a *linear equation*.

Mark scheme

C = 35h + 60

1 mark for 35h and 1 mark for + 60

c) A company records the number of sick days taken by its employees over the past three months. The data is recorded below.

0	3	2	2	0
8	5	5	2 3	1
1	8	2 6 5	7	2
2	1	6	1	2 2 3
4	4	5	1	3
0	0	4	3 6	1
2	3	5	6	1
4 2	0	0	4	0
2	3	6	1	2

i) State the number of employees recorded.

Mark scheme

45 (1 mark)

Questions continue on the next page

ii) Construct a suitable *frequency table* for the data.

Mark scheme

Number of sick days	Frequency
0	7
1	8
2	9
3	6
4	5
5	4
6	3
7	1
8	2

Correct format (1 mark) Correct frequency values (1 mark)

(Total 2 marks)

iii) Calculate the *sample mean* of the data.

Mark scheme

Number of sick days x	Frequency f	fx
0	7	0
1	8	8
2	9	18
3	6	18
4	5	20
5	4	20
6	3	18
7	1	7
8	2	16

Correct fx column (1 mark) $\sum f = 45$

 $\sum fx = 125$ (1 mark)

Sample mean: $\overline{x} = \frac{\sum fx}{\sum f} = \frac{125}{45}$ (1 mark) = 2.8 (to 1 d.p.) (1 mark)

(Total 4 marks)

Total 20 Marks

4

Question 22

a) In an experiment, Reena rolls a seven-sided spinner, numbered 1 to 7, 630 times. The results are recorded in the table below.

Score	1	2	3	4	5	6	7
Frequency	65	64	80	140	73	155	53

i) Calculate the *relative frequency* of the spinner landing on a 7.

Mark scheme

Relative frequency of landing on 7: $\frac{53}{630}$ (1 mark)

ii) Calculate the *relative frequency* of the spinner landing on a 2 or a 7.

2

1

Mark scheme

Relative frequency of landing on a 7: $\frac{53}{630}$ from i) above. Relative frequency of landing on a 2: $\frac{64}{630}$ (1 mark) Relative frequency of landing on a 1 or a 2: $\frac{53}{630} + \frac{64}{630} = \frac{117}{630} = \frac{13}{70}$ or 0.19 (to 2 d.p.) (1 mark for correct answer, simplified or not)

(Total 2 marks)

iii) Find the *theoretical probability* of a fair, seven-sided spinner landing on a 7.
2 State the number of times you would expect a fair, seven-sided spinner to land on a 7 in a trial of 630 spins.

Mark scheme

The theoretical probability of a fair, seven - sided spinner landing on a 7 is: P (landing on a 7) = $\frac{1}{7}$ (1 mark)

The number of times that a fair, seven-sided spinner would be expected to land on a 7 in a trial of 630 throws is: $\frac{1}{7} \times 630 = 90$ (1 mark)

(Total 2 marks)

iv) State whether you think that the spinner used in Reena's experiment is fair. Give a reason for your answer.

Mark scheme

No, I do not think the Spinner is fair (1 mark) The relative frequency of landing on a 1 (2, 3, 5 or 7) is lower than the theoretical probability, whilst the relative frequencies of landing on a 4 or 6 are higher than the theoretical probability. Therefore, it is likely that the spinner is not fair (valid reason, 1 mark).

If the candidate thinks that the spinner is fair, award full marks if they provide a suitable argument.

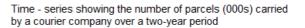
b) A courier company records the number of parcels, in thousands (000s), carried over a two year period.

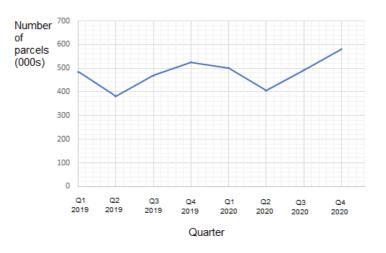
Year		2	019			20	20	
Quarter	1	2	3	4	1	2	3	4
Number of parcels (000s)	485	381	470	525	501	405	492	580

i) Draw a *time series graph* for this data using the graph paper provided.

6

Mark scheme





- 1 mark for each correctly drawn and labelled axis (2 marks)
- Points plotted correctly (3 marks) NB. Deduct 1 mark for each error, up to a maximum of 3 marks
 Points joined by a straight line (1 mark)

(Total 6 marks)

- Marks
- ii) The first three four-point averages for the data are 465.25, 469.25 and 475.25 Calculate the fourth and fifth four-point moving averages.

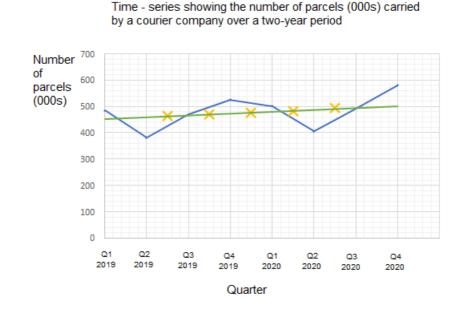
Mark Scheme

 $(525+501+405+492) \div 4 = 480.75$ (1 mark) $(501+405+492+580) \div 4 = 494.5$ (1 mark)

(Total 2 marks)

iii) Plot the moving averages on the time series graph constructed in i) and draw **3** the trendline.

Mark scheme



- Moving averages plotted correctly (up to 2 marks). NB. Deduct 1 mark for each error up to a maximum of 2 marks.
- Trendline (1 mark)

(Total 3 marks)

- iv) Use the graph to comment upon *seasonality* and *trend*.
 - The data suggests there is a high season in Q4 and a low season in Q2 (1 mark)
 - The trendline indicates that the number of parcels carried by the courier increasing (1 mark)

Total 20 Marks

Question 23

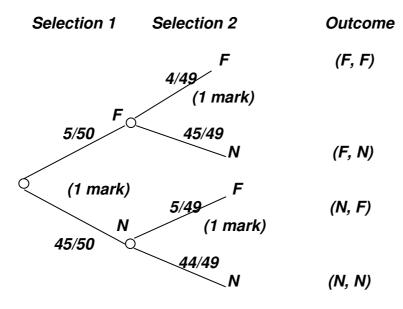
- **a)** A factory produces 50 laptops . FIVE (5) of the laptops are faulty. TWO (2) laptops s are selected at random without replacement.
 - i) Draw a tree diagram to show all the possible outcomes.

4

2

Mark scheme

Suggested tree diagram below where F denotes a faulty shirt and N denotes a laptop with no faults.



3 marks for correct branches, 1 mark for correct outcomes. Total 4 marks

ii) Calculate the probability that the first laptop selected is faulty and the second **2** laptop selected is not.

Mark scheme

 $P(FN) = \frac{5}{50} \times \frac{45}{49} = \frac{9}{98}$

(1 mark for working, 1 mark for correct answer, simplified or not.)

iii) Calculate the probability that both laptops are faulty.

Mark scheme

 $P(FF) = \frac{5}{50} \times \frac{4}{49} = \frac{2}{245}$

(1 mark for working, 1 mark for correct answer, simplified or not.)

iv) Calculate the probability that both laptops selected are either faulty or not faulty. **3**

Mark scheme

$$P(both faulty or both not faulty) = P(FF) + P(NN) = \frac{5}{50} \times \frac{4}{49} + \frac{45}{50} \times \frac{44}{49} = \frac{2}{245} + \frac{198}{245} = \frac{40}{49}$$

(1 mark partially correct working, 2 marks for wholly correct working, 1 mark for correct answer, simplified or not.)

v) Calculate the probability that at least one of the laptops selected will not be faulty.

Mark scheme

 $P(at \ least \ one \ shirt \ not \ faulty) = 1 - P(FF) \ P(at \ least \ one \ laptop \ not \ faulty) = 1 - P(FF) \}$

$$=1-\frac{2}{245}$$

$$=\frac{243}{245}$$

d) Using the quadratic formula, find the exact solutions of:

 $2x^2 - 5x + 1 = 0.$

Then, give your answers to 2 d.p.

Mark scheme

 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 2, \ b = -5, \ c = 1$ $x = \frac{5 \pm \sqrt{(-5)^2 - (4 \times 2 \times 1)}}{2 \times 2}$ (correct substitution, 1 mark) $= \frac{5 \pm \sqrt{17}}{4}$

x = 2.28 (to 2 d. p.) and x = 0.22 (to 2 d. p.)

Method (1 mark) Correct answers (1 mark)

(Total 3 marks)

b) Factorise completely: $x^2 - 14x + 45$

Mark scheme

$$(x-9)(x-5)$$

1 mark for (x - 9)1 mark for (x - 5)

(Total 2 marks)

c) Expand and simplify: (x+4)(x+3)

Mark scheme

 $x^2 + 7x + 12$

Correct equation (2 marks) Deduct 1 mark per error, up to a maximum of 2 marks.

(Total 2 marks)

Total 20 Marks

2

Question 24

a) An online company want to know how frequently customers purchase products up to £250. Data is recorded in the table below.

Price range, p, £	Frequency
$0 \le p < 50$	13
50 ≤ <i>p</i> < 100	16
100 ≤ <i>p</i> < 150	24
150 ≤ <i>p</i> < 200	29
$200 \le p < 250$	18

i) Complete the *cumulative frequency* column for the data.

Price range, p, \mathfrak{L}	Frequency	Cumulative frequency
$0 \le p < 50$	13	
50 ≤ <i>p</i> < 100	16	
100 ≤ <i>p</i> < 150	24	
150 ≤ <i>p</i> < 200	29	
200 ≤ <i>p</i> < 250	18	

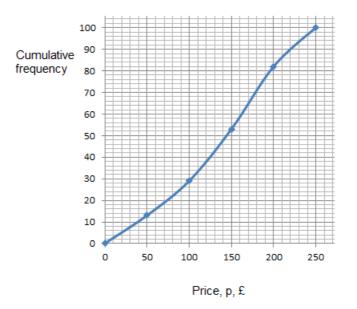
Mark scheme

Price range, p, £	Frequency	Cumulative frequency
0 ≤ <i>p</i> < 50	13	13
50 ≤ p < 100	16	29
100 ≤ p < 150	24	53
150 ≤ p < 200	29	82
200 ≤ p < 250	18	100

Award 2 marks for correctly completed cumulative frequency column – deduct 1 mark for any error up to maximum of 2 marks.

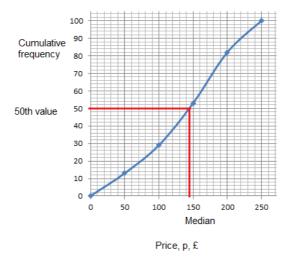
ii) Plot a cumulative frequency graph for the data using the graph paper provided.

Mark scheme



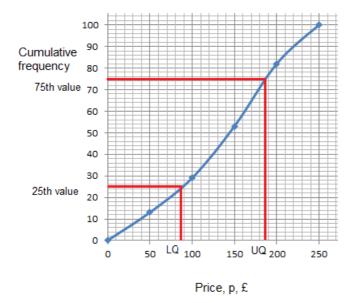
- Cumulative frequency diagram with marks along the horizontal axis and cumulative frequency on vertical axis. Axis scale should be clear and appropriate. (2 marks)
- Points correctly plotted at upper end of class interval e.g. (50, 13)
- (2 marks, 1 mark deducted for error)
- Points joined in a smooth curve starting at the origin (1 mark)
- iii) Use your cumulative frequency graph to estimate the *median*. You should **2** show how you found the *median* on your graph.

Mark scheme



- Accept values of the median between 142 and 148
- Award 1 mark for some indication of how median has been found using graph and 1 mark for value of median.
- iv) Use your cumulative frequency graph to estimate the values of the *lower* quartile and the *upper quartile*. You should show how you have found the *lower quartile* and *upper quartile* on your graph.

Mark scheme



- Accept value of lower quartile between 82 and 90 (1 mark)
- Award 1 mark for some indication of how lower quartile has been found using graph.
- Accept value of upper quartile between 182 and 190 (1 mark)
- Award 1 mark for some indication of how upper quartile has been found using graph

v) Calculate the *interquartile range*. Show your workings.

Mark scheme

Interquartile range = upper quartile – lower quartile

The actual value of the interquartile range will vary depending on the values of the lower quartile and upper quartile found in (iv). However, to be awarded the mark, it should be clear that the candidate has subtracted the lower quartile from the upper quartile.

For example: 187 – 87 = 100 (1 mark)

b) State whether the data collected in part (a) is *quantitative* or *qualitative*. Briefly explain your answer.

Mark scheme

Quantitative (1 mark) The data is numeric (1 mark)

c) A cafe records the number of customers each day. The results for the past two weeks are shown in the table below:

	Week 1			Week 2								
Day	М	Tu	W	Th	F	S	М	Tu	W	Th	F	S
No. of	65	58	69	***	78	91	63	61	72	75	82	90
customers												

i) Explain why it is appropriate to calculate a six-point moving average for this data.

Mark scheme

The data has been collected 6 times a week (1 mark)

ii) The value of the first six-point moving average is 72. Calculate the number **3** of customers on Thursday of week ONE (1).

Mark scheme

 $(65+58+69+x+78+91) \div 6 = 72$ x+361 = 432So, x = 71

- 2 marks for wholly correct method
- 1 mark for partially correct method
- Correct answer (1 mark)

Total 20 Marks

End of paper

Learning Outcomes matrix

Question	Learning Outcomes assessed	Marker can differentiate between varying levels of achievement
1-20	All	Yes
21	1, 2	Yes
22	3, 4	Yes
23	1, 3	Yes
24	2, 4	Yes

Grade descriptors

Learning Outcome	Fail	Referral	Pass	Merit	Distinction
Use a range of mathematical and statistical techniques and concepts	Can basically identify, adapt and use appropriate skills, methods and procedures to reach basic solutions.	In a limited way, can identify, adapt and use appropriate skills, methods and procedures to reach limited solutions.	Can adequately Identify, adapt and use appropriate skills, methods and procedures to reach appropriate solutions.	Can soundly identify, adapt and use appropriate skills, methods and procedures to reach supported and appropriate solutions.	Can coherently identify, adapt and use appropriate skills, methods and procedures to reach well supported and highly appropriate solutions.
Describe and summarise data	Demonstrates basic ability to review the effectiveness and appropriatenes s of actions, methods and results	Demonstrates limited ability to review the effectiveness and appropriatenes s of actions, methods and results	Demonstrates adequate ability to review the effectiveness and appropriateness of actions, methods and results	Demonstrates sound ability to review the effectiveness and appropriateness of actions, methods and results	Demonstrates comprehensive ability to review the effectiveness and appropriateness of actions, methods and results
Apply the laws of probability to a range of scenarios	Provides basic interpretation and evaluation of relevant information and ideas to address problems that are well defined but non-routine.	Provides limited interpretation and evaluation of relevant information and ideas to address problems that are well defined but non-routine.	Provides consistent interpretation and evaluation of relevant information and ideas to address problems that are well defined but non-routine.	Provides critical interpretation and evaluation of relevant information and ideas to address problems that are well defined but non-routine.	Provides consistently critical interpretation and evaluation of relevant information and ideas to address problems that are well defined but non-routine.
Use data for the purposes of forecasting	Can basically identify, adapt and use appropriate skills, methods and procedures to reach basic solutions.	In a limited way, can identify, adapt and use appropriate skills, methods and procedures to reach limited solutions.	Can adequately Identify, adapt and use appropriate skills, methods and procedures to reach appropriate solutions.	Can soundly identify, adapt and use appropriate skills, methods and procedures to reach supported and appropriate solutions.	Can coherently identify, adapt and use appropriate skills, methods and procedures to reach well supported and highly appropriate solutions.