Foundation Mathematics

Sample

Marking Scheme

This marking scheme has been prepared as a **guide only** to markers. This is not a set of model answers, or the exclusive answers to the questions, and there will frequently be alternative responses which will provide a valid answer. Markers are advised that, unless a question specifies that an answer be provided in a particular form, then an answer that is correct (factually or in practical terms) **must** be given the available marks.

If there is doubt as to the correctness of an answer, the relevant NCC Education materials should be the first authority.

Throughout the marking, please credit any valid alternative point.

Where markers award half marks in any part of a question, they should ensure that the total mark recorded for the question is rounded up to a whole mark.

Marks

Question 1

a) Simplify the following:

i) $r^9 \div r^4 \div r^2$ **1**

r^3

ii) $m^5 \times m^3 \times m^{-4}$ **1**

m^4

iii) $p^2 q \times p^4 q^{-3}$ **1**

$p^6 q^{-2}$

b) Simplify the following:

i) $\dfrac{4}{3xy} \times \dfrac{x^2}{3y^3}$ **1**

$\dfrac{4x}{9y^4}$

ii) $\dfrac{24a^4 b^2 c}{6a^3 c}$ **1**

$4ab^2$

iii) $r(3r^2 - s) + s(r - r^4)$ **1**

$3r^3 - r^4 s$

c) Factorise the following:

i) $12x^2 y + 6xy^3$ **2**

$6xy(2x + y^2)$

1 mark for correct working, 1 mark for correct answer.

ii) $u^2 + 10u + 21$ **2**

$(u + 7)(u + 3)$

1 mark for correct working, 1 mark for correct answer.

d) Simplify the following:

i) $\dfrac{5}{3x} + \dfrac{1}{4x}$ **2**

$$= \dfrac{20 + 3}{12x}$$

$$= \dfrac{23}{12x}$$

1 mark for correct working, 1 mark for correct answer.

ii) $\dfrac{7}{15b} - \dfrac{2}{5b}$ **2**

$$= \dfrac{7 - 6}{15b}$$

$$= \dfrac{1}{15b}$$

1 mark for correct working, 1 mark for correct answer.

e) Transpose the following formula to make x the subject: **2**
$$y = \dfrac{x(z + 2)}{7}$$

$$x = \dfrac{7y}{(z + 2)}$$

1 mark for correct working, 1 mark for correct answer.

f) Solve the following equation **and** find the value of g: **2**
$$3g - 10 = 8$$

$$g = 6$$

1 mark for correct working, 1 mark for correct answer.

g) Solve the following quadratic equation by factorising: **2**
$$x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0$$
So, $x = -8$ or $x = 2$

1 mark for correct working, 1 mark for correct answer.

Total 20 Marks

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Question 2

a) Solve the following quadratic equation by using the quadratic formula: **2**
$$5x^2 + 11x - 3 = 0$$
You may leave your answer in surd form.

$$x = -\frac{11 \pm \sqrt{181}}{10}$$

1 mark for correct working, 1 mark for correct answer.

b) Solve the following simultaneous equations and find the value of x and y.

i) $3x + 8y = 2$ and $5x - 8y = -18$ **2**
$$x = -2, y = 1$$

1 mark for correct working, 1 mark for correct answer.

ii) $6x - 2y = 8$ and $2x + y = 11$ **2**
$$x = 3, y = 5$$

1 mark for correct working, 1 mark for correct answer.

c) Tom thinks of a number, n. Tom adds 3 to it and then multiplies the answer by 4   **3**
and gets 32.
What is the value of n?
$$4(n + 3) = 32$$
$$n + 3 = \frac{32}{4}$$
$$n = \frac{32}{4} - 3$$
$$n = 5$$

Up to 2 marks for correct workings, 1 mark for correct answer.

d) Calculate the gradient of the following curves using differentiation at the point where $x = 2$.

i) $y = 3x^2 - 5x$ **3**
Differentiating gives:
$$\frac{dy}{dx} = 6x - 5$$

When $x = 2$:
$$\frac{dy}{dx} = 6(2) - 5$$
$$= 7$$

So when $x = 2$ the gradient is 7.

Up to 2 marks for correct differentiation of curve, 1 mark for correct gradient.

ii) $y = x^3 + 0.25x^2$ **3**
Differentiating gives:
$$\frac{dy}{dx} = 3x^2 + 0.5x$$

When $x = 2$:
$$\frac{dy}{dx} = 3(2)^2 + 0.5(2)$$
$$= 13$$

So when $x = 2$ the gradient is 13.

Up to 2 marks for correct differentiation of curve, 1 mark for correct gradient.

e) A particle has a velocity of $v = 5t^3 - 10t$.

     i)   Find the acceleration, a, after t seconds.   **2**

$$a = \frac{dv}{dt} = 15t^2 - 10$$

1 mark for correct working, 1 mark for correct answer.

     ii)   What is the acceleration at $t = 3$ seconds?   **1**
When $t = 3$:

$$a = 15t^2 - 10$$
$$= 15(3)^2 - 10$$
$$= 125 \text{ m/s}^2$$

     iii)   At what time, t, is the acceleration 50 m/s²?   **2**
Substitute $a = 50$ into the equation for acceleration to give:
$$50 = 15t^2 - 10$$
$$15t^2 = 60$$
$$t^2 = \frac{60}{15} = 4$$
$$t = 2 \text{ (since } t \text{ cannot be 2)}$$
Hence the acceleration is 50 m/s² at $t = 2$ seconds.

1 mark for correct working, 1 mark for correct answer.

Total 20 Marks

Question 3

a) i)   Using differentiation, find the coordinates of the turning point on the curve: **4**
$$y = -3x^2 - 3x + 6$$
Differentiating gives:
$$\frac{dy}{dx} = -6x - 3$$
Turning points are located where:
$$\frac{dy}{dx} = -6x - 3 = 0$$
$$\frac{dy}{dx} = 0 \text{ when } x = -0.5$$
When $x = -0.5$, $y = 6.75$
So, the turning point is at (-0.5, 6.75).

Up to 2 marks for correctly differentiating the curve, 1 mark for working and 1 mark for correct coordinates.

ii) Construct and complete a table of values for the curve $-3x^2 - 3x + 6$ for
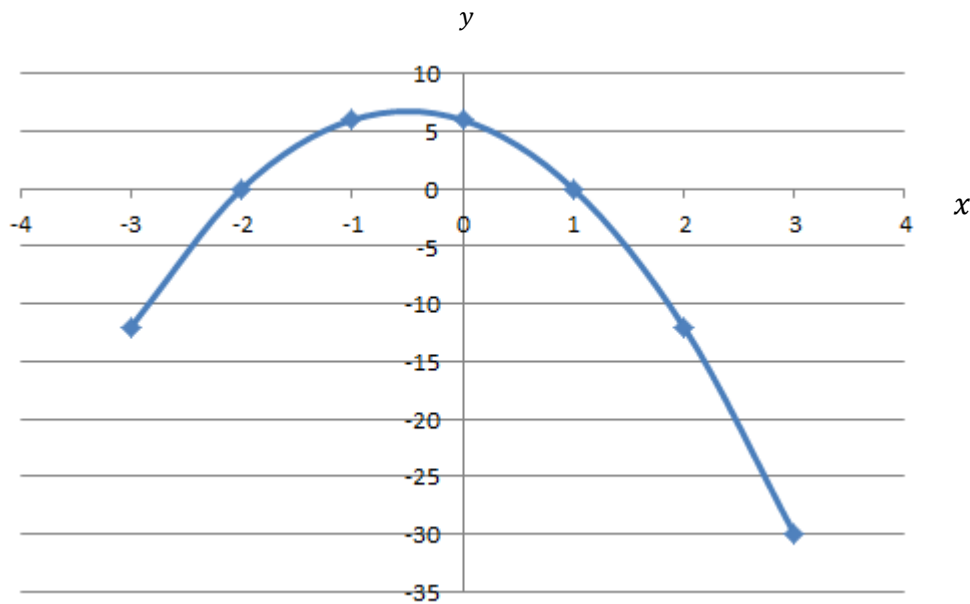 $-3 \leq x \leq 3$.

Use these values to plot a graph of the curve $y = -3x^2 - 3x + 6$ and identify
the turning point found in part (a)(i) as either a maximum or minimum turning
point. You should use the graph paper provided.

Table of values

x	-3	-2	-1	0	1	2	3
$y = -3x^2 - 3x + 6$	-12	0	6	6	0	-12	-30

2 marks for table of values.

Graph of the curve $y = -3x^2 - 3x + 6$

2 marks for correctly plotting curve.

Hence the turning point at (-0.5, 6.75) is a maximum turning point.

1 mark for correct classification of turning point.

b) Integrate the following expression:

$$3x^5 + \sqrt{x}$$

$$\int \left(3x^5 + \sqrt{x}\right) dx = \int \left(3x^5 + x^{1/2}\right) dx$$
$$= \frac{1}{2}x^6 + \frac{2}{3}x^{3/2} + c$$

where c is a constant.

1 mark for correct working, 1 mark for correct answer.

2

c) The gradient of the curve which passes through the point (2, 17) is given by $6x^2 + 4x$. Find the equation of the curve. 3

The gradient is given by $6x^2 + 4x$ hence $\frac{dy}{dx} = 6x^2 + 4x$

Integrate to obtain $y = \int (6x^2 + 4x)\,dx$
$$= 2x^3 + 2x^2 + c$$
The curve passes through the point (2, 17), so when $x = 2$, $y = 17$
Substitute these values back into the integrated equation to obtain:
$$17 = 2(2)^3 + (2)^2 + c$$
$$= 24 + (-7)$$
So, $c = -7$
Therefore the equation of the curve is $y = 2x^3 + 2x^2 - 7$

Up to 2 marks for correct working, 1 mark for correct answer.

d) Evaluate the definite integral: 3

$$\int_1^3 (8x^3 - x)\,dx$$

$$\int_1^3 (8x^3 - x)\,dx = \left[\frac{8x^4}{4} - \frac{x^2}{2}\right]_1^3$$
$$= \left[2x^4 - \frac{1}{2}x^2\right]_1^3$$
$$= \left\{2(3)^4 - \frac{1}{2}(3)^2\right\} - \left\{2(1)^4 - \frac{1}{2}(1)^2\right\}$$
$$= 156$$

Up to 2 marks for correct working, 1 mark for correct answer.

e) Find the area bounded by the curve $y = 7x^3 + 3x^2$, the x-axis and the lines $x = 1$ and $x = 2$. 3

$$A = \int_1^2 (7x^3 + 3x^2)\,dx$$
$$= \left[\frac{7x^4}{4} + x^3\right]_1^2$$
$$= \left\{\frac{7(2)^4}{4} + (2)^3\right\} - \left\{\frac{7(1)^4}{4} + (1)^3\right\}$$
$$= 36 - 2.75$$
$$= 33.25\ units$$

Up to 2 marks for correct workings, 1 mark for correct answer.

<div style="text-align:right">**Total 20 Marks**</div>

Question 4

a) The velocity of a moving body is $3t^2 + 5$ metres per second after a time of **3**
t seconds. Find the distance travelled by the body at the end of 3 seconds.
Distance is given by the equation $s = \int_0^3 v\,dt$

Integrate the equation for v using the limits of 0 and 3. Assume $s = 0$ when
$t = 0$ therefore $c = 0$.

$$s = \int_0^3 (3t^2 + 5)\,dt$$
$$= \left[\frac{3t^3}{3} + 5t\right]_0^3$$
$$= [t^3 + 5t]_0^3$$
$$= \{(3)^3 + 5(3)\} - \{0\}$$
$$= \{42\} - \{0\}$$
$$= 42m$$

Up to 2 marks for correct working, 1 mark for correct answer.

b) Reena and Simon are playing a game of chess and a game of draughts against
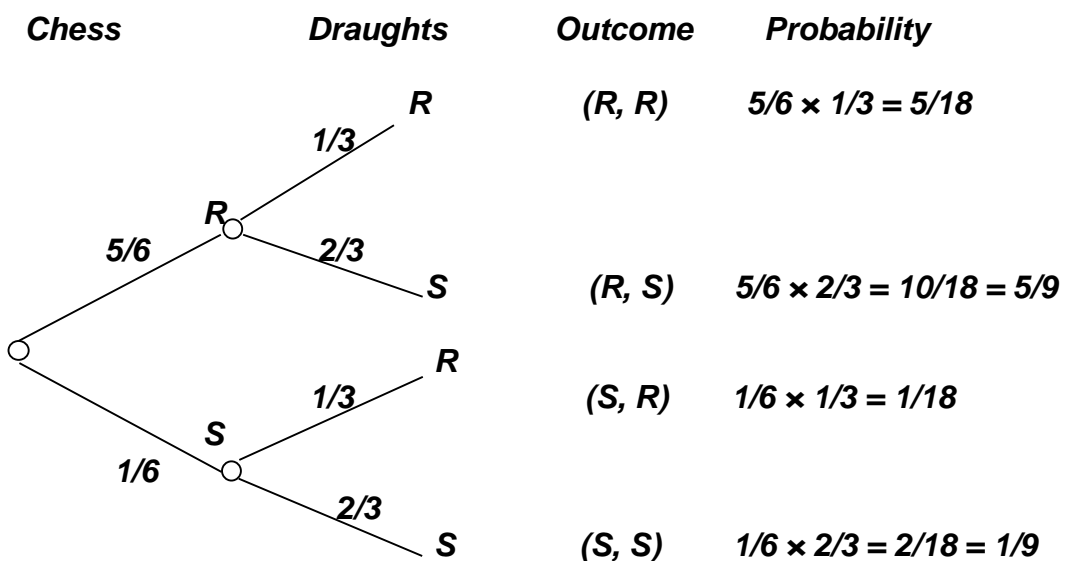each other.
The probability that Reena will win the chess game is $\frac{5}{6}$.
The probability that Simon will win the draughts game is $\frac{2}{3}$.
There are no draws in either game.

   i) Draw a probability tree diagram to show all the possible outcomes. **8**
   Draw a tree diagram, writing 'R' for a Reena win and 'S' for a Simon
   win.

Chess	Draughts	Outcome	Probability
	R 1/3	(R, R)	5/6 × 1/3 = 5/18
R 5/6 2/3	S	(R, S)	5/6 × 2/3 = 10/18 = 5/9
	R 1/3	(S, R)	1/6 × 1/3 = 1/18
S 1/6 2/3	S	(S, S)	1/6 × 2/3 = 2/18 = 1/9

   1 mark for correct workings for each branch, 1 mark for each correct
   outcome probability

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ii) Use your tree diagram to find out the probability that Reena wins one game **2**
and Simon wins one game.

$$P(Reena\ wins\ one\ game\ and\ Simon\ wins\ one\ game) = P(R,S) + P(S,R)$$
$$= \frac{10}{18} + \frac{1}{18}$$
$$= \frac{11}{18}$$

1 mark for correct working, 1 mark for correct answer. If only incorrect due to errors in part (i), award full marks.

iii) Use your tree diagram to find out the probability that one person wins both **2**
games.

$$P(one\ person\ wins\ both\ games) = P(R,R) + P(S,S)$$
$$= \frac{5}{18} + \frac{2}{18}$$
$$= \frac{7}{18}$$

1 mark for correct working, 1 mark for correct answer. If only incorrect due to errors in part (i), award full marks.

iv) Use your tree diagram to find the probability that Simon wins at least one **2**
game.

The only outcome whereby Simon does not win at least one game is if Reena wins both games.

$$P(Simon\ wins\ at\ least\ one\ game) = 1 - P(R,R)$$
$$= 1 - \frac{5}{18}$$
$$= \frac{13}{18}$$

1 mark for correct working, 1 mark for correct answer. If only incorrect due to errors in part (i), award full marks.

c) You are given a box containing 7 books. How many ways can you arrange 3 of **3**
them on a shelf?
This is a permutation since order is important.

$${}^7P_3 \frac{n!}{(n-r)!} = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210$$

Up to 2 marks for correct working, 1 mark for correct answer.

Total 20 Marks

Question 5

a) A college records the grades achieved by 40 students in a science exam. The results are shown in the table below.

Method of transport of 60 shoppers.

Grade	Frequency
Distinction	7
Merit	11
Pass	18
Fail	4

The college decides to present the data as a pie chart.

i) Calculate the relative frequency for 'Distinction' **and** the angle of the 'Distinction' sector on the pie chart. **2**
   'Distinction' relative frequency:
   $\frac{7}{40} = 17.5\%$

   1 mark for correct value.

   'Distinction' angle of sector:
   $\frac{7}{40} \times 360° = 63°$

   1 mark for correct value.

ii) Calculate the relative frequency for 'Merit' **and** the angle of the 'Merit' sector on the pie chart. **2**
   'Merit' relative frequency:
   $\frac{11}{40} = 27.5\%$

   1 mark for correct value.

   'Merit' angle of sector:
   $\frac{11}{40} \times 360° = 99°$

   1 mark for correct value.

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iii) Calculate the relative frequency for 'Pass' **and** the angle of the 'Pass' sector
on the pie chart.
'Pass' relative frequency:
$$\frac{18}{40} = 45\%$$

1 mark for correct value.

'Pass' angle of sector:
$$\frac{18}{40} \times 360° = 162°$$

1 mark for correct value.

b) A hotel manager records the age of people staying in the hotel on one particular
night as follows:

Age (years)	Frequency
0 < 15	32
15 < 30	13
30 < 45	20
45 < 60	17
60 < 75	15
75 < 90	8

i) Calculate the mean age of the people. **4**

Age (years)	Midpoint	Frequency f	fx
0 < 15	7.5	32	240
15 < 30	22.5	13	292.5
30 < 45	37.5	20	750
45 < 60	52.5	17	892.5
60 < 75	67.5	15	1012.5
75 < 90	82.5	8	660
		$\Sigma f = 105$	$\Sigma fx = 3847.5$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{3847.5}{105}$$
$$= 36.6$$

Up to 3 marks for correct working, 1 mark for correct answer.

ii) Construct a histogram to illustrate the data from part (b)(i).

A histogram showing the ages of people staying in a hotel on one night.
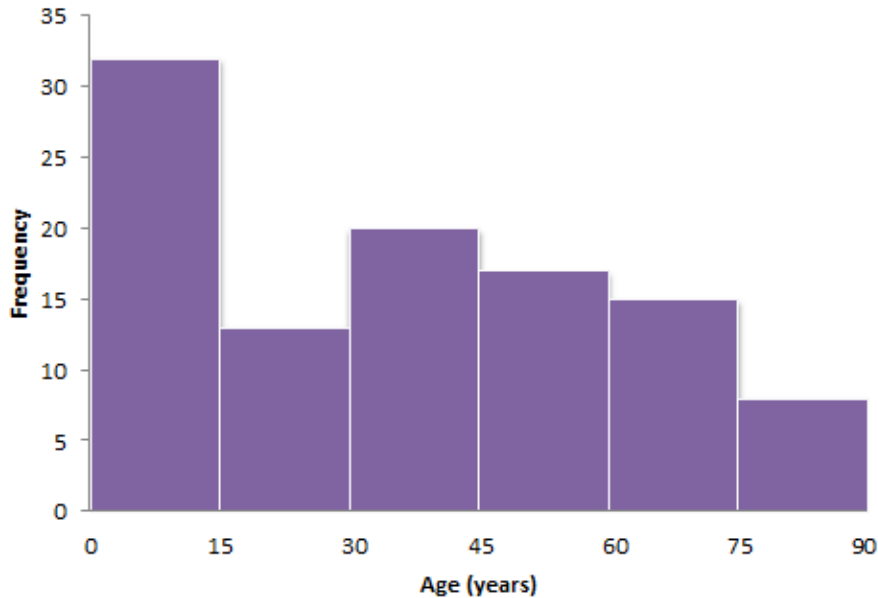
2 marks for histogram with age on horizontal axis and frequency on vertical axis. 2 marks for accurate plotting of bars.

c) The following data set is recorded:

51 24 37 39 42 31 38 29 53 59 22

i) Calculate the range of the data. **1**
59 - 22 = 37

ii) Find the median of the data. **2**
Put the data into ascending order:
22 24 29 31 37 38 39 42 51 53 59

The median value is the:
$$\frac{(n+1)}{2} = \frac{12}{2} = 6th\ value$$
So the median is 38.

1 mark for workings, 1 mark for correct answer.

iii) Find the lower quartile and the upper quartile of the data. **2**

The lower quartile is:
$$\frac{1}{4}(n+1) = \frac{1}{4}(12) = 3rd\ value$$
So, the lower quartile is 29.

The upper quartile is:
$$\frac{3}{4}(n+1) = \frac{3}{4}(12) = 9th\ value$$
So, the upper quartile is 51.

1 mark for correct lower quartile value, 1 mark for correct upper quartile value.

iv) Calculate the quartile range of the data. **1**
The quartile range is given by:
upper quartile – lower quartile = 51 – 29 = 22

Total 20 Marks

End of paper

Learning Outcomes matrix

Question	Learning Outcomes assessed	Marker can differentiate between varying levels of achievement
1	1, 2	Yes
2	2, 4	Yes
3	4, 5	Yes
4	5, 7	Yes
5	2, 3, 6	Yes