Further Mathematics

SAMPLE

Examination Paper

Answer ALL questions.

Clearly cross out surplus answers.

Time: 2 hours

The maximum mark for this paper is 100.

Any reference material brought into the examination room must be handed to the invigilator before the start of the examination.

Candidates are allowed to use a scientific calculator during this examination.

Graph paper will be provided by the centre.

<u>You must show your workings.</u>
Marks are awarded for these.

Marks

Question 1

a) Use the factor theorem to show that $(x - 4)$ is a factor of $f(x) = 2x^3 - x^2 - 25x - 12$ — **2**

b) Hence, or otherwise, fully factorise $f(x)$. — **3**

c) Show that $\sum_{r=1}^{n} 4r^2 + 6r + 1 = \frac{n}{3}[4n^2 + 15n + 12]$ — **4**

d) Given that 3 and $4 + i$ are roots of the cubic equation $f(x) = x^3 - 11x^2 + ax + b = 0$

   i) Find the value of a and the value of b — **4**

   ii) Show the three roots on an Argand diagram — **2**

e) Given $f(x) = \frac{3(9x^2+1)}{(3x+1)(3x-1)} \equiv A + \frac{B}{3x+1} + \frac{C}{3x-1}$,

   i) Find the values of A, B and C — **3**

   ii) Use the quotient rule to differentiate $f(x)$, giving your answer in the form $f'(x) = \frac{-Qx}{(9x^2-1)^2}$ — **2**

Total 20 Marks

Question 2

a) A curve has equation $f(x) = \frac{5}{(1+x)(3-2x)}$

   i) Express $f(x)$ in terms of its partial fractions. — **4**

b)
   i) Use the binomial theorem to expand $f(x)$ in ascending powers of x, up to and including the term in x^3, simplifying each term. — **5**

   ii) Determine the range of values of x for which the expansion is valid. — **1**

c) Use your answer to a) to find $f'(x)$ and hence find the x – coordinate of the stationary point of $f(x)$ — **5**

d) Write down the equations of all the asymptotes of f(x) — **2**

e) Sketch the graph of $f(x)$ — **3**

Total 20 Marks

Question 3

a) Given $Z_1 = 4 + \sqrt{2}i$ and $\frac{Z_1}{Z_2} = 3 + i$, find:

i) Z_2 in the form $a + ib$ **5**

ii) The argument of Z_2 **2**

iii) The exact value of $|Z_2|$ **3**

b) Given $Z_3 = -4 + 3i$ and $Z_4 = 3 + 2i$, find

i) The modulus of $(Z_4)(Z_3)^2$ **4**

ii) Show on an Argand diagram the points A and B, where A represents Z_3 and B represents Z_4 **2**

iii) Find $Z_3 Z_4$ in polar form **2**

iv) Determine angle $A\hat{O}B$ **1**

v) Write Z_4 in exponential form **1**

Total 20 Marks

Question 4

a) Prove that $cosh^2(x) - sinh^2(x) \equiv 1$ using the exponential definition for $cosh(x)$ and $sinh(x)$ **2**

b) Show that the equation $tan^2(x) = 3\sec(x) + 9 = 0$ can be written as $sec^2(x) - 3\sec(x) - 10 = 0$ **2**

c) Hence, solve $tan^2(x) = 3\sec(x) + 9$ giving all values of x in the interval $0 \leq x \leq 360$ giving solutions to 2 d.p. where necessary **4**

d) Sketch the graph of $y = 3\sinh(2x) + 2$ **2**

e) Prove that $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$ **4**

f) Find the first three terms in the series expansion of $cos(x)$ using Maclaurin's expansion **2**

i) Given $(1 + x)^{-1} \approx 1 - x + x^2$, find the first three non-zero terms in the expansion of sec(x), in ascending order. **4**

Total 20 Marks

Question 5

a)
Describe the transformation represented by the matrix $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

2

    i)   Find A^2

2

    ii) Describe the transformation represented by A^2

1

b) Given $A = \begin{pmatrix} a & -6 \\ 2 & a+6 \end{pmatrix}$

    i)   Find det(A) giving your answer in terms of a.

2

    ii) Show that the matrix A is non-singular for all values of a.

2

    iii) Find A^{-1}, when a=3

2

c) A rectangular hyperbola H has parametric equations given by $x = 3t, y = \frac{3}{t}, t \neq 0$.

    i)   The line L has equation $6y = 4x - 15$. Show that L intersects H when $4t^2 - 5t - 6 = 0$

3

    ii) Find $\frac{dy}{dx}$ at the point where t=2

3

    ii) The line intersects the hyperbola at two points, a and b. Find the coordinates of a and the coordinates of b.

3

Total 20 Marks

End of paper