Level 3 International Foundation Diploma for Higher Education Studies
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Further Mathematics for University Study

SAMPLE EXAMINATION

This specimen assessment material is devised from previous past papers to reflect the single examination assessment approach now taken for this unit in the Level 3 International Foundation Diploma for Higher Education Studies.

Time allowed: 2 hours

Marking Scheme

This marking scheme has been prepared as a guide only to markers. This is not a set of model answers, or the exclusive answers to the questions, and there will frequently be alternative responses which will provide a valid answer. Markers are advised that, unless a question specifies that an answer be provided in a particular form, then an answer that is correct (factually or in practical terms) must be given the available marks.

If there is doubt as to the correctness of an answer, the relevant NCC Education materials should be the first authority.

Throughout marking, please credit any valid alternative point.

Candidate Name and ID number:		
Marker's comments:		
Moderator's comments:		
Mark:	**Moderated mark:**	**Final mark:**
Penalties applied for academic malpractice:		

Answer ALL questions

Marks

Question 1

a) Differentiate each of the following with respect to x, giving the answer in its simplest form:

i) $f(x) = (2x^3 - 4x^2)(3x^5 + x^2)$ **3**

Mark Scheme:

Use the product rule:

$$\boldsymbol{f'(x) = u'v + uv'}$$

Let:

$$\boldsymbol{u = 2x^3 - 4x^2, \quad v = 3x^5 + x^2}$$

$$\boldsymbol{u' = 6x^2 - 8x, \quad v' = 15x^4 + 2x}$$

$$\boldsymbol{f'(x) = (6x^2 - 8x)\left(3x^5 + x^2\right) + (2x^3 - 4x^2)(15x^4 + 2x)}$$

[M1]

Expand:

$$\boldsymbol{(6x^2 - 8x)\left(3x^5 + x^2\right) = 18x^7 + 6x^4 - 24x^6 - 8x^3 =}$$

$$\boldsymbol{(2x^3 - 4x^2)(15x^4 + 2x) = 30x^7 + 4x^4 - 60x^6 - 8x^3}$$

[M1]

Combine:

$$\boldsymbol{f'(x) = 24x^7 - 60x^6 + 10x^4 - 16x^3}$$

[A1]

Marks

ii) $g(x) = \dfrac{2x^3 + 4}{x^2 - 4x + 1}$ **4**

Mark Scheme:

Quotient rule:

$$\boldsymbol{g'(x) = \frac{vu' - uv'}{v^2}}$$

Let:

$$\boldsymbol{u = 2x^3 + 4, \quad u' = 6x^2, \quad v = x^2 - 4x + 1, \quad v' = 2x - 4}$$

[M1]

$$\boldsymbol{g'(x) = \frac{(x^2 - 4x + 1)(6x^2) - (2x^3 + 4)(2x - 4)}{(x^2 - 4x + 1)^2}}$$

[M1]

Expand:

$$\boldsymbol{6x^2(x^2 - 4x + 1) = 6x^4 - 24x^3 + 6x^2}$$

$$\boldsymbol{(2x^3 + 4)(2x - 4) = 4x^4 - 8x^3 + 8x - 16}$$

Subtract:

$$\boldsymbol{2x^4 - 16x^3 + 6x^2 - 8x + 16}$$

[M1]

Final answer:

$$\boldsymbol{g'(x) = \frac{2x^4 - 16x^3 + 6x^2 - 8x + 16}{(x^2 - 4x + 1)^2}}$$

[A1]

iii) $h(x) = ln(sinx)$ **2**

Mark Scheme:

Use chain rule:

$$\mathrm{h}^{'(x)} = \frac{1}{\sin x} \cdot \cos x$$

[M1]

$$h'(x) = \cot x$$

[A1]

Marks

b) Find $\frac{dy}{dx}$ for the equation: **3**

$$x^2 + y = 1 + y^3$$

Mark Scheme:

Differentiate both sides:

$$\mathbf{2x + \frac{dy}{dx} = 3y^2\frac{dy}{dx} \Rightarrow}$$

[M2 – 1 mark for each side correctly differentiated]

$$\mathbf{2x = \frac{dy}{dx}(3y^2 - 1) \Rightarrow}$$

$$\mathbf{\frac{dy}{dx} = \frac{2x}{3y^2 - 1}}$$

[A1]

c) Evaluate the following integral: **4**

$$\int_0^1 \frac{3}{\sqrt{4 - x^2}}dx$$

Mark Scheme:

Recognise standard form:

$$\mathbf{\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right), a = 2}$$

[M1]

$$\mathbf{\int_0^1 \frac{3}{\sqrt{4 - x^2}}dx = 3\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1}$$

[M1]

$$\mathbf{3\left(\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)\right) = 3\left(\frac{\pi}{6} - 0\right)}$$

[M1]

$$\mathbf{= \frac{\pi}{2}}$$

[A1]

Marks

d) Estimate: **4**

$$\int_1^2 \ln x \, dx, \text{ using the trapezium rule with } n = 5$$

State ONE (1) way that the estimation can be made more accurate.

Mark Scheme:

Step size:

$$\boldsymbol{h = \frac{2-1}{5} = 0.2}$$

[M1]

x-values: $\mathbf{1, 1.2, 1.4, 1.6, 1.8, 2}$

Corresponding y-values:

$$\boldsymbol{y_0 = ln1 = 0, \quad y_1 = ln1.2, \quad y_2 = ln1.4, \quad \dots, \quad y_5 = ln2}$$

Apply rule:

$$\boldsymbol{\int_1^2 ln\, x\, dx \approx \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3 + y_4) + y_5] \approx 0.1(3.152 + 0.693)}$$
$$\boldsymbol{= 0.1(3.845)}$$

[M1]

$$\boldsymbol{= 0.3845}$$

[A1]

For more accuracy we could take more divisions of the interval so the trapezia more closely follows the shapes of the curve.

[A1]

Total 20 Marks

Marks

Question 2

a) Given the triangle PQR with coordinates: **3**

$$P(1,1), \quad Q(3,1), \quad R(4,4)$$

State the coordinate area formula and use this to calculate the area of triangle PQR.

Mark Scheme:
Use of the coordinate area formula:

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

[M1]

$$= \frac{1}{2}|1(1-4) + 3(4-1) + 4(1-1)|$$

[M1]

$$= \frac{1}{2}|-3 + 9 + 0| = \frac{1}{2}(6) = 3$$

[A1]

b) The coordinates of A, B, C and D are as follows: **3**

$$A(2,3), \quad B(6,5), \quad C(-1,-4), \quad D(3,-2)$$

Show that segment AB is **parallel** to segment CD.

Mark Scheme:
Gradient of AB:

$$m_{AB} = \frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$$

[M1]

Gradient of CD:

$$m_{CD} = \frac{-2-(-4)}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

[M1]

Since both gradients are equal:

$$AB \parallel CD$$

[A1]

Marks

c) Plot the TWO (2) graphs of this system over the domain $-5 \leq x \leq 5$. **8**

Solve algebraically for x:

$$\begin{cases} 3x - y = 2 \\ x^2 - y = 5 \end{cases}$$

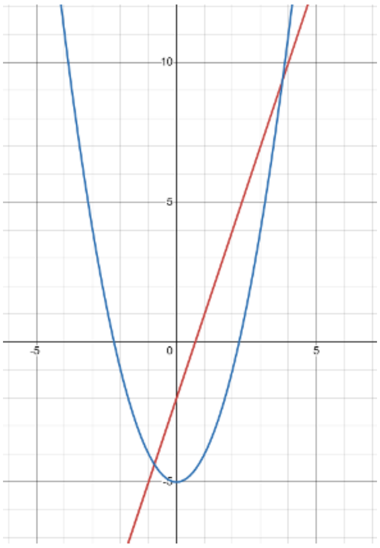
You **must** use graph paper in this question.

Mark Scheme:
Rearrange to:

$$y = 3x - 2 \quad \text{and} \quad y = x^2 - 5$$

[M1]

Plot both graphs over the domain $-5 \leq x \leq 5$. ***Identify their points of intersection.***



[M1 – graphs have smooth curve and straight lines]
[M1 – graphs have x and y axis clearly labelled]
[M2 – 1 mark per graph correct]

Set $3x - 2 = x^2 - 5$:

$$x^2 - 3x - 3 = 0$$

Use the quadratic formula:

$$x = \frac{3 \pm \sqrt{(-3)^2 + 4 \cdot 1 \cdot 3}}{2} = \frac{3 \pm \sqrt{21}}{2}$$

[M1]

Approximate solutions:

$$x \approx -0.79,\ y \approx -4.37 \quad \text{and} \quad x \approx 3.79,\ y \approx 9.37$$

[A2 – 1 mark per set of solutions]

Marks

d) A lighthouse is located at point $A(2,5)$, and a buoy is placed at point $B(8,-1)$ in a harbour.

A circular safety zone is created such that AB is the diameter of the circle.

i) Find the coordinates of the centre of the circle. **2**

Mark Scheme:
Midpoint of AB:

$$C = \left(\frac{2+8}{2}, \frac{5+(-1)}{2}\right)$$

[M1]

$$= (5,2)$$

[A1]

ii) Find the radius of the circle. **2**

Mark Scheme:
Radius = half the length of AB:

$$\text{Length of AB} = \sqrt{(8-2)^2 + (-1-5)^2}$$

[M1]

$$\sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$\text{Radius} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

[A1]

iii) Write the equation of the circle in its simplest form. **2**

Mark Scheme:
Equation of the circle:

$$(x-5)^2 + (y-2)^2 = \left(3\sqrt{2}\right)^2 = 18$$

[A2 –2 marks for all correct, allow M1 mark if small error made in calculation]

Total 20 Marks

Marks

Question 3

a) Given: **4**

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ -3 & 0 \\ 5 & 2 \end{bmatrix}$$

Find the product AB.

Mark Scheme:

$$\boldsymbol{AB} = \begin{bmatrix} (\mathbf{1})(\mathbf{2}) + (\mathbf{0})(-\mathbf{3}) + (\mathbf{3})(\mathbf{5}) & (\mathbf{1})(\mathbf{1}) + (\mathbf{0})(\mathbf{0}) + (\mathbf{3})(\mathbf{2}) \\ (\mathbf{2})(\mathbf{2}) + (-\mathbf{1})(-\mathbf{3}) + (\mathbf{4})(\mathbf{5}) & (\mathbf{2})(\mathbf{1}) + (-\mathbf{1})(\mathbf{0}) + (\mathbf{4})(\mathbf{2}) \end{bmatrix} = \begin{bmatrix} \mathbf{17} & \mathbf{7} \\ \mathbf{27} & \mathbf{10} \end{bmatrix}$$

[M4 – 4 marks for fully correct answer, deduct 1 mark for each quadrant error]

b) Given:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 3 & -1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

i) Find $\det(A)$ using Sarrus' Rule. **3**

Mark Scheme:

Downward diagonals:

$$2(-1)(2) + 1(1)(1) + 0(3)(1) = -4 + 1 + 0 = -3$$

[M1]

Upward diagonals:

$$1(-1)(0) + 0(1)(2) + 2(3)(1) = 0 + 0 + 6 = 6$$

[M1]

$$det(A) = -3 - 6 = -9$$

[A1]

Marks

ii) Find A^{-1} **4**

Mark Scheme:

$$\boldsymbol{det(A) - 9}$$

Cofactors (not all shown here):

$$\textbf{\textit{Cofactor matrix}} = \begin{pmatrix} -2 & -5 & 1 \\ 2 & 4 & -1 \\ 1 & -2 & -5 \end{pmatrix}$$

[M1]

$$\Rightarrow \textbf{\textit{Adjugate}} = \begin{pmatrix} -2 & 2 & 1 \\ -5 & 4 & -2 \\ 1 & -1 & -5 \end{pmatrix}$$

[M1]

$$A^{-1} = \frac{1}{-9}\begin{pmatrix} -2 & 2 & 1 \\ -5 & 4 & -2 \\ 1 & -1 & -5 \end{pmatrix}$$

[M1]

$$= \begin{pmatrix} \frac{2}{9} & -\frac{2}{9} & -\frac{1}{9} \\ \frac{5}{9} & -\frac{4}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{1}{9} & \frac{5}{9} \end{pmatrix}$$

[A1]

Marks

c) Solve the following system of equations using Cramer's rule: **5**

$$\begin{cases} -x + 2y - z = 2 \\ -x + y = -3 \\ 3x + 2y - 2z = -2 \end{cases}$$

Mark Scheme:

$$\boldsymbol{A} = \begin{pmatrix} \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{3} & \mathbf{2} & \mathbf{-2} \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} \mathbf{2} \\ \mathbf{-3} \\ \mathbf{-2} \end{pmatrix}$$

[M1]

$$\boldsymbol{det}(\boldsymbol{A}) == \mathbf{-1(-2) - 2(2) - 1(-2-3) = 2 - 4 + 5 = 3}$$

$$\boldsymbol{det}(\boldsymbol{A}) = \mathbf{3}$$

[M1]

Replace each column in A with B and calculate determinants.

Then compute:

$$\boldsymbol{x} = \frac{\mathbf{-12}}{\mathbf{3}} = \mathbf{-4}, \quad \boldsymbol{y} = \frac{\mathbf{-21}}{\mathbf{3}} = \mathbf{-7}, \quad \boldsymbol{z} = \frac{\mathbf{-36}}{\mathbf{3}} = \mathbf{-12}$$

[M3 – 1 for each correct value]

Marks
4

d) Given:

$$T = \begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\sqrt{2} & 2\sqrt{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Find the volume of the unit sphere after transformation and state if the sphere maintains orientation.

Mark Scheme:

Original volume:

$$\boldsymbol{V = \frac{4}{3}\pi}$$

$$\boldsymbol{det(T) = 3 \cdot \left[\sqrt{2}\left(2\sqrt{2}\right) - \sqrt{2}\left(-\sqrt{2}\right)\right]}$$

[M1]

$$\boldsymbol{= 3(4 + 2) = 18 \Rightarrow}$$

[M1]

$$\boldsymbol{V_{new} = 18 \cdot \frac{4}{3}\pi = 24\pi}$$

[A1]

$\boldsymbol{det(T) > 0 \Rightarrow}$ ***orientation is preserved***

[A1]

Total 20 Marks

Marks

Question 4

a) Express each of the following complex numbers in the form:

$$z = r(\cos\theta + i\sin\theta)$$

where r is the modulus and θ the argument (in radians).

i) $z = 1 + \sqrt{3}i$ **3**

Mark Scheme:

$$\boldsymbol{r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = \sqrt{4} = 2}$$

[M1]

$$\boldsymbol{\theta = tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}}$$

[M1]

$$\boldsymbol{z = 2\left(cos\left(\frac{\pi}{3}\right) + isin\left(\frac{\pi}{3}\right)\right)}$$

[A1]

ii) $z = 1 - 2i$ **3**

Mark Scheme:

$$\boldsymbol{\sqrt{1^2 + (-2)^2} = \sqrt{5}, \quad arg(z) = \tan^{-1}(-2)}$$

[M2]

$$\boldsymbol{z = \sqrt{5}(cos(-1.107) + isin(-1.107))}$$

[A1]

Marks

b) Find all the cube roots of 8 giving your answers in the form $z = a + bi$. **3**

Mark Scheme:

$$\mathbf{8 = 8(\cos 0 + i\sin 0)}$$

[M1]

$$\boldsymbol{z_k = 2\left(\cos\left(\frac{2k\pi}{3}\right) + i\sin\left(\frac{2k\pi}{3}\right)\right), \quad k = 0, 1, 2}$$

[M1]

$$\boldsymbol{z_0 = 2, \quad z_1 = -1 + i\sqrt{3}, \quad z_2 = -1 - i\sqrt{3}}$$

[A1]

c) The inequality:

$$2 < |z - (1 - 2i)| < 5$$

describes a region on an Argand diagram.

i) Describe geometrically what this region represents. **1**

Mark Scheme:

Ring / Annulus

[A1]

ii) State the centre, inner radius and outer radius. **2**

Mark Scheme:

- ***Inner radius: 2, outer radius: 5***
- ***around point*** $\mathbf{(1, -2)}$

[A2 – 1 for each bullet]

iii) Find the area of the region. **1**

Mark Scheme:

$$\boldsymbol{A = \pi(5^2 - 2^2) = \pi(25 - 4) = 21\pi}$$

[A1]

Marks

d) Let:

$$z = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$$

Express z^4 in:

i) Trigonometric form. **2**

Mark Scheme:

$$\boldsymbol{z^4 = 2^4\left(\cos\left(\frac{4\pi}{6}\right) + i\sin\left(\frac{4\pi}{6}\right)\right)}$$

[M1]

$$\boldsymbol{= 16\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)}$$

[A1]

ii) Exponential form. **1**

Mark Scheme:

$$\boldsymbol{z^4 = 16e^{i\frac{2\pi}{3}}}$$

[A1]

iii) Cartesian form. **1**

Mark Scheme:

$$\boldsymbol{z^4 = 16\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -8 + 8i\sqrt{3}}$$

[A1]

Marks

e) Given that: **3**

$$z_1 = 3 + 4i, \quad z_2 = 1 - 2i$$

Compute $z_1 \cdot \overline{z_2} - \overline{z_1} \cdot z_2$.

Mark Scheme:

$$\overline{\boldsymbol{z_1}} = \mathbf{3 - 4}\boldsymbol{i}, \quad \overline{\boldsymbol{z_2}} = \mathbf{1 + 2}\boldsymbol{i}$$

$$\boldsymbol{z_1} \cdot \overline{\boldsymbol{z_2}} = (\mathbf{3 + 4}\boldsymbol{i})(\mathbf{1 + 2}\boldsymbol{i}) = \mathbf{-5 + 10}\boldsymbol{i}$$

[M1]

$$\overline{\boldsymbol{z_1}} \cdot \boldsymbol{z_2} = (\mathbf{3 - 4}\boldsymbol{i})(\mathbf{1 - 2}\boldsymbol{i}) = \mathbf{-5 - 10}\boldsymbol{i}$$

[M1]

$$\Rightarrow (\mathbf{-5 + 10}\boldsymbol{i}) - (\mathbf{-5 - 10}\boldsymbol{i}) = \mathbf{20}\boldsymbol{i}$$

[A1]

Total 20 Marks

Marks

Question 5

a) Show that $\cosh^2 x - \sinh^2 x = 1$ using exponential definitions. **3**

Mark Scheme:
Use:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

[M1]

$$\cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 =$$

$$= \frac{e^{2x} + e^{-2x} + 2}{4} - \frac{e^{2x} + e^{-2x} - 2}{4}$$

[M1]

$$\frac{4}{4} = 1$$

[A1]

b) Evaluate $\tanh(\ln 2)$ exactly. **2**

Mark Scheme:

$$\tanh(\ln 2) = \frac{2 - \frac{1}{2}}{2 + \frac{1}{2}}$$

[M1]

$$= \frac{3/2}{5/2} = \frac{3}{5}$$

[A1]

c) Express $\sinh^{-1}(3)$ in logarithmic form. **1**

Mark Scheme:

$$\sinh^{-1}(3) = \ln\left(3 + \sqrt{3^2 + 1}\right) = \ln\left(3 + \sqrt{10}\right)$$

[M1]

Marks

d) Consider the expression $y = \tan x$.

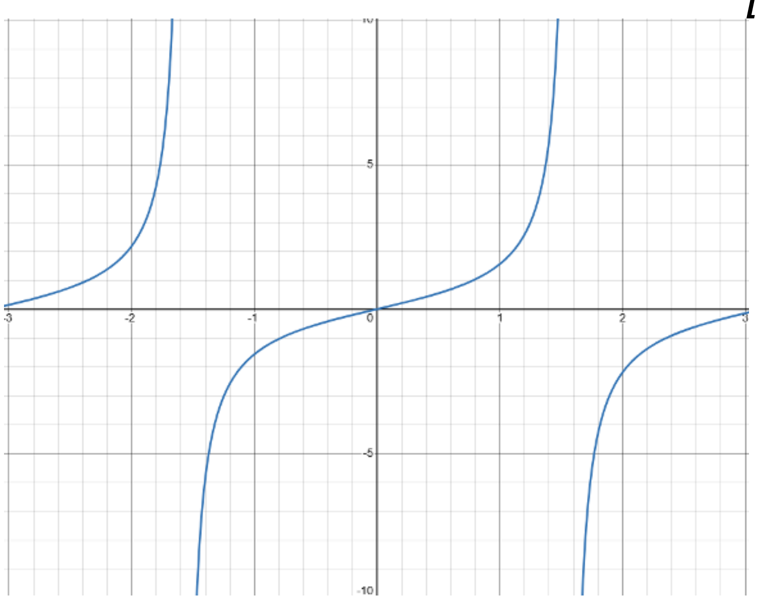
i) Sketch the graph of $y = \tan x$ for: **4**

$$x \in [-\pi,\ \pi]$$

Mark Scheme:

- ***Asymptotes at*** $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2},$
- ***Zeros at*** $x = -\pi, 0, \pi$
- ***Shape of graph is correct***
- ***Range and labels are correct***

[M4 -1 mark per bullet]



ii) State the domain and range of $\tan x$. **2**

Mark Scheme:

Domain: $x \in \mathbb{R} \backslash \left\{(2k+1)\frac{\pi}{2} \mid k \in \mathbb{Z}\right\}$

[A1]

Range: $y \in \mathbb{R}$

[A1]

iii) Determine algebraically whether $\tan x$ is an odd function. **2**

Mark Scheme:

$$\tan(-x) = \frac{-\sin x}{\cos x} = -\tan x$$

[M1]

$\Rightarrow$ odd function

[A1]

Marks

e) An equation T is given as:

$$\sin\left(x+\frac{\pi}{6}\right)+\sin\left(x-\frac{\pi}{6}\right)=1 \quad \text{for } 0\le x\le\pi$$

i) Prove that: **2**

$$\sin(A+B)+\sin(A-B)=2\sin A\cos B$$

Mark Scheme:

$$\begin{aligned}\sin(A+B) &= \sin A\cos B+\cos A\sin B\\ \sin(A-B) &= \sin A\cos B-\cos A\sin B\\ \sin(A+B)+\sin(A-B) &= (\sin A\cos B+\cos A\sin B)+(\sin A\cos B-\cos A\sin B)\\ &= 2\sin A\cos B\end{aligned}$$

[M1] [A1]

ii) Solve the equation T, giving your answers to 3 decimal places. **4**

Mark Scheme:

$$\Rightarrow \boldsymbol{2sinx\cdot cos\left(\frac{\pi}{6}\right)=1}\Rightarrow$$

[M1]

$$\boldsymbol{\sqrt{3}sinx=1}$$

$$\Rightarrow \boldsymbol{sinx=\frac{\sqrt{3}}{3}}$$

[M1]

$$\boldsymbol{x=sin^{-1}\left(\frac{\sqrt{3}}{3}\right)\approx 0.615 \textit{ or } \pi-0.615\approx 2.527}$$

[A2 – 1 mark per answer to 3dp]

Total 20 Marks

End of paper

NCC Further Mathematics Formula Sheet

Trigonometric identities

$\sin(A \pm B) \equiv \sin(A)\cos(B) \pm \cos(A)\sin(B)$
$\cos(A \pm B) \equiv \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\tan(A \pm B) \equiv \dfrac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)} \quad \left(A \pm B \neq \left(k + \dfrac{1}{2}\right)\pi\right)$

Differentiation

f(x)	f'(x)
$\tan(kx)$	$k\sec^2(kx)$
$\sec(x)$	$\sec(x)\tan(x)$
$\cot(x)$	$-\mathrm{cosec}^2(x)$
$\mathrm{cosec}(x)$	$-\mathrm{cosec}(x)\cot(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\mathrm{sech}^2(x)$

Summation

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{matrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{matrix}$

Reflection in the line y = tan(θ): $\begin{matrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{matrix}$

Complex Numbers

$e^{i\theta} = \cos(\theta) + i\sin(\theta)$
$[r(\cos(\theta) + i\sin(\theta))]^n = r^n(\cos(n\theta) + i\sin(n\theta))$
The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \dots, n-1$

Hyperbolic Functions

$\cosh^2(x) - \sinh^2(x) \equiv 1$

$\sinh(2x) \equiv 2\sinh(x)\cosh(x)$

$\cosh(2x) \equiv \cosh(x)^2 + \sinh(x)^2$

$\cosh^{-1}(x) \equiv \ln\left(x + \sqrt{x^2 - 1}\right)\ (x \geq 1)$

$\sinh^{-1}(x) \equiv \ln\left(x + \sqrt{x^2 + 1}\right)$

$\tanh^{-1}(x) \equiv \frac{1}{2}\ln\left(\frac{1+x}{1+x}\right) z\ (|x| < 1)$

Conic

	Ellipse	Parabola	Hyperbola	Rectangular Hyperbola
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos(\theta), b\sin(\theta))$	$(at^2, 2at)$	$(a\sec(\theta), b\tan(\theta))$ $(\pm a\cosh(\theta), b\sinh(\theta))$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	None	None	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Maclaurin's and Taylor's Series

$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^r}{r!}f^{(r)}(0) + \cdots$

$f(x) = f(a) + (x - a)\,f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots + \frac{(x-a)^r}{r!}f^{(r)}(a) + \cdots$

$f(a + x) = f(a) + x\,f'(a) + \frac{x^2}{2!}f''(a) + \cdots + \frac{x^r}{r!}f^{(r)}(a) + \cdots$

$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} + \cdots$ for all x

$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{r+1}x^r}{r} + \cdots\ (-1 < x \leq 1)$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \cdots$ for all x

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^r x^{2r}}{(2r)!} + \cdots$ for all x

$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + \frac{(-1)^r x^{2r+1}}{(2r+1)} + \cdots\ (-1 \leq x \leq 1)$

Learning Outcomes Matrix

Question	Learning Outcomes / Assessment Criteria assessed	Marker can differentiate between varying levels of achievement
1	LO5	Yes
2	LO2	Yes
3	LO6	Yes
4	LO8	Yes
5	LO4	Yes

Grade Descriptors

Learning Outcome	Pass	Merit	Distinction
1. Recognise and employ principles of algebra and how it is an essential tool that supports and expresses mathematical reasoning and provides a means to generalise across a number of contexts.	Demonstrate adequate understanding of techniques	Demonstrate robust understanding of techniques	Demonstrate highly comprehensive understanding of techniques
2. Recognise and work with coordinate geometry and identify how algebraic representations also describe a spatial relationship, which gives us a new way to understand a situation.	Demonstrate ability to perform the tasks	Demonstrate ability to perform the tasks consistently well	Demonstrate ability to perform the tasks to the highest standard
3. Recognise, employ principles of and work effectively with sequence and series.	Demonstrate ability to perform techniques	Demonstrate ability to perform techniques consistently well	Demonstrate ability to perform techniques to the highest standard
4. Recognise, identify, and work with the different types of equations and functions including trigonometric, logarithmic and exponential functions/	Demonstrate adequate understanding of techniques	Demonstrate robust understanding of techniques	Demonstrate highly comprehensive understanding of techniques
5. Identify, interpret and use the techniques in calculus to perform differentiation and integration on individual and combinations of different types of functions and how to use these techniques to solve problems involving functions given parametrically.	Demonstrate adequate understanding of techniques	Demonstrate robust understanding of techniques	Demonstrate highly comprehensive understanding of techniques
6. Recognise and work effectively with vectors and matrices.	Demonstrate ability to perform the tasks	Demonstrate ability to perform the tasks consistently well	Demonstrate ability to perform the tasks to the highest standard
7. Recognise how to solve equations numerically.	Demonstrate ability to perform techniques	Demonstrate ability to perform techniques consistently well	Demonstrate ability to perform techniques to the highest standard
8. Recognise, employ the principles of and work effectively with differential equations.	Demonstrate adequate understanding of techniques	Demonstrate robust understanding of techniques	Demonstrate highly comprehensive understanding of techniques

Learning Outcome	Pass	Merit	Distinction
9. Be able to effectively work with complex numbers, perform arithmetic calculations using complex numbers, solve higher order polynomials with complex roots and sketch regions in the complex plane.	Demonstrate ability to perform techniques	Demonstrate ability to perform techniques consistently well	Demonstrate ability to perform techniques to the highest standard