Level 3 International Foundation Diploma for Higher Education Studies
Ofqual Number: 603/0080/2   Quartz ID: 1125 / 1207

Component ID: 2368

Further Mathematics for University Study

SAMPLE EXAMINATION

This specimen assessment material is devised from previous past papers to reflect the single examination assessment approach now taken for this unit in the Level 3 International Foundation Diploma for Higher Education Studies.

Time allowed: 2 hours

Instructions:
- Answer **ALL** questions.
- Clearly cross out surplus answers.
- You must show your workings. Marks are awarded for these.
- Please refer to the formula sheet at the end of this examination paper.
- Any reference material brought into the examination room must be handed to the invigilator before the start of the examination.

Information:
- The maximum mark for this paper is 100.
- Graph paper will be provided by the centre.
- **You may use a scientific calculator during this examination.**
- Note: Scientific calculators permitted in the exam should not be able to carry out algebraic manipulation, differentiation or integration, language translation or communication with other devices of the internet. They should not have a QWERTY keyboard. There should not be retrievable information stored in the calculator, for example databanks, dictionaries, mathematical formulas or text.

Answer ALL questions

Marks

Question 1

a) Differentiate each of the following with respect to x, giving the answer in its simplest form:

i) $f(x) = (2x^3 - 4x^2)(3x^5 + x^2)$ **3**

ii) $g(x) = \dfrac{2x^3 + 4}{x^2 - 4x + 1}$ **4**

iii) $h(x) = ln(sinx)$ **2**

b) Find $\frac{dy}{dx}$ for the equation: **3**

$$x^2 + y = 1 + y^3$$

c) Evaluate the following integral: **4**

$$\int_0^1 \frac{3}{\sqrt{4 - x^2}} dx$$

d) Estimate: **4**

$$\int_1^2 \ln x \, dx, \text{ using the trapezium rule with } n = 5$$

State ONE (1) way that the estimation can be made more accurate.

Total 20 Marks

Marks

Question 2

a) Given the triangle PQR with coordinates: **3**

$$P(1,1), \quad Q(3,1), \quad R(4,4)$$

State the coordinate area formula and use this to calculate the area of triangle PQR.

b) The coordinates of A, B, C and D are as follows: **3**

$$A(2,3), \quad B(6,5), \quad C(-1,-4), \quad D(3,-2)$$

Show that segment AB is **parallel** to segment CD.

c) Plot the TWO (2) graphs of this system over the domain $-5 \leq x \leq 5$. **8**

Solve algebraically for x:

$$\begin{cases} 3x - y = 2 \\ x^2 - y = 5 \end{cases}$$

You **must** use graph paper in this question.

d) A lighthouse is located at point $A(2,5)$, and a buoy is placed at point $B(8,-1)$ in a harbour.

A circular safety zone is created such that AB is the diameter of the circle.

i) Find the coordinates of the centre of the circle. **2**

ii) Find the radius of the circle. **2**

iii) Write the equation of the circle in its simplest form. **2**

Total 20 Marks

Marks

Question 3

a) Given: **4**

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ -3 & 0 \\ 5 & 2 \end{bmatrix}$$

Find the product AB.

b) Given:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 3 & -1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

i) Find $\det(A)$ using Sarrus' Rule. **3**

ii) Find A^{-1} **4**

c) Solve the following system of equations using Cramer's rule: **5**

$$\begin{cases} -x + 2y - z = 2 \\ -x + y = -3 \\ 3x + 2y - 2z = -2 \end{cases}$$

d) Given: **4**

$$T = \begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\sqrt{2} & 2\sqrt{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Find the volume of the unit sphere after transformation and state if the sphere maintains orientation.

Total 20 Marks

Marks

Question 4

a) Express each of the following complex numbers in the form:

$$z = r(\cos\theta + i\sin\theta)$$

where r is the modulus and θ the argument (in radians).

i) $z = 1 + \sqrt{3}i$ **3**

ii) $z = 1 - 2i$ **3**

b) Find all the cube roots of 8 giving your answers in the form $z = a + bi$. **3**

c) The inequality:

$$2 < |z - (1 - 2i)| < 5$$

describes a region on an Argand diagram.

i) Describe geometrically what this region represents. **1**

ii) State the centre, inner radius and outer radius. **2**

iii) Find the area of the region. **1**

d) Let:

$$z = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$$

Express z^4 in:

i) Trigonometric form. **2**

ii) Exponential form. **1**

iii) Cartesian form. **1**

Marks

e) Given that: **3**

$$z_1 = 3 + 4i, \quad z_2 = 1 - 2i$$

Compute $z_1 \cdot \overline{z_2} - \overline{z_1} \cdot z_2$.

Total 20 Marks

Marks

Question 5

a) Show that $\cosh^2 x - \sinh^2 x = 1$ using exponential definitions. **3**

b) Evaluate $\tanh(\ln 2)$ exactly. **2**

c) Express $\sinh^{-1}(3)$ in logarithmic form. **1**

d) Consider the expression $y = \tan x$.

i) Sketch the graph of $y = \tan x$ for: **4**

$$x \in [-\pi,\ \pi]$$

ii) State the domain and range of $\tan x$. **2**

iii) Determine algebraically whether $\tan x$ is an odd function. **2**

e) An equation T is given as:

$$\sin\left(x + \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{6}\right) = 1 \quad \text{for } 0 \leq x \leq \pi$$

i) Prove that: **2**

$$\sin(A + B) + \sin(A - B) = 2\sin A\cos B$$

ii) Solve the equation T, giving your answers to 3 decimal places. **4**

Total 20 Marks

End of paper

NCC Further Mathematics Formula Sheet

Trigonometric identities

$\sin(A \pm B) \equiv \sin(A)\cos(B) \pm \cos(A)\sin(B)$
$\cos(A \pm B) \equiv \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\tan(A \pm B) \equiv \dfrac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$

Differentiation

f(x)	f'(x)
$\tan(kx)$	$k\sec^2(kx)$
$\sec(x)$	$\sec(x)\tan(x)$
$\cot(x)$	$-\mathrm{cosec}^2(x)$
$\mathrm{cosec}(x)$	$-\mathrm{cosec}(x)\cot(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\mathrm{sech}^2(x)$

Summation

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{matrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{matrix}$

Reflection in the line y = tan(θ): $\begin{matrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{matrix}$

Complex Numbers

$e^{i\theta} = \cos(\theta) + i\sin(\theta)$
$[r(\cos(\theta) + i\sin(\theta))]^n = r^n(\cos(n\theta) + i\sin(n\theta))$
The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \dots, n-1$

Hyperbolic Functions

$\cosh^2(x) - \sinh^2(x) \equiv 1$

$\sinh(2x) \equiv 2\sinh(x)\cosh(x)$

$\cosh(2x) \equiv \cosh(x)^2 + \sinh(x)^2$

$\cosh^{-1}(x) \equiv \ln\left(x + \sqrt{x^2 - 1}\right) \ (x \geq 1)$

$\sinh^{-1}(x) \equiv \ln\left(x + \sqrt{x^2 + 1}\right)$

$\tanh^{-1}(x) \equiv \frac{1}{2}\ln\left(\frac{1+x}{1+x}\right) z \ (|x| < 1)$

Conic

	Ellipse	Parabola	Hyperbola	Rectangular Hyperbola
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos(\theta), b\sin(\theta))$	$(at^2, 2at)$	$(a\sec(\theta), b\tan(\theta))$ $(\pm a\cosh(\theta), b\sinh(\theta))$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	None	None	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Maclaurin's and Taylor's Series

$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \cdots + \frac{x^r}{r!} f^{(r)}(0) + \cdots$

$f(x) = f(a) + (x - a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots + \frac{(x-a)^r}{r!} f^{(r)}(a) + \cdots$

$f(a + x) = f(a) + x f'(a) + \frac{x^2}{2!} f''(a) + \cdots + \frac{x^r}{r!} f^{(r)}(a) + \cdots$

$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} + \cdots \text{ for all } x$

$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{r+1}x^r}{r} + \cdots (-1 < x \leq 1)$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \cdots \text{ for all } x$

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^r x^{2r}}{(2r)!} + \cdots \text{ for all } x$

$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + \frac{(-1)^r x^{2r+1}}{(2r+1)} + \cdots (-1 \leq x \leq 1)$

