Level 3 International Foundation Diploma for Higher Education Studies
Ofqual Number: 600/0080/2 Quartz ID: 1125 / 1207

Component ID: 2367

Mathematics for University Study

SAMPLE EXAMINATION

This specimen assessment material is devised from previous past papers to reflect the single examination assessment approach now taken for this unit in the Level 3 International Foundation Diploma for Higher Education Studies.

Time allowed: 1 hour 30 minutes

Instructions:

- Answer **ALL** questions.
- Clearly cross out surplus answers.
- You must show your workings. Marks are awarded for these.
- Please refer to the formula sheet at the end of this examination paper.
- Any reference material brought into the examination room must be handed to the invigilator before the start of the examination.

Information:

- The maximum mark for this paper is 100.
- Graph paper will be provided by the centre.
- **You may use a scientific calculator during this examination.**
- Note: Scientific calculators permitted in the exam should not be able to carry out algebraic manipulation, differentiation or integration, language translation or communication with other devices of the internet. They should not have a QWERTY keyboard. There should not be retrievable information stored in the calculator, for example databanks, dictionaries, mathematical formulas or text.

Answer ALL questions

Marks

Question 1

a) A certain laptop in the UK costs £605. The same laptop in Spain costs €680 Euros. The exchange rate from £ to Euros is £1:1.15 Euros.

i) Determine in which country the laptop is more expensive. **2**

ii) In the US, the same laptop costs $810. The exchange rate from £ to dollars is £1:$1.35. Calculate the value of the US laptop in Pounds Sterling (£). **2**

iii) Ava is from the US. Say where you would recommend that she buys the laptop from – the US, Spain, or the UK? Provide a reason. **2**

b) Ravi's final year grade depends on three modules, totalling 100 credits. Module 1 is worth 30 credits, Module 2 is worth 30 credits, and Module 3 is worth 40 credits.

i) Express how much the THREE (3) modules contribute towards the final year grade as a ratio. **1**

ii) Ravi is aiming to achieve a final year grade of 80%. If he has already achieved marks of 88% and 68% for Modules 1 and 2, what mark does Ravi require for Module 3 to achieve his aim? **4**

c) Saira leaves home at 10:00. After 1 hour and 25 minutes of moving at a constant speed, she has travelled 65 miles. She then stops for some lunch. After stopping for 1 hour, she drives to another location a further 25 miles away at a constant speed. After driving 45 minutes, she stops at this location. After stopping here for 35 minutes, she drives towards home at a constant speed. It takes her 1.5 hours to get home.

i) Construct a distance-time graph using the graph paper provided, to show this information. **5**

ii) There are FIVE (5) parts to Saira's journey. Find her speed (in miles per hour) in each of the FIRST, THIRD, AND FIFTH parts. **3**

d) A car is travelling at 15 metres per second (m/s). It slows down to 5 m/s in 10 seconds. **1**

Kit incorrectly says that the deceleration of the car was 10 m/s.

Give ONE (1) correction to this statement.

Total 20 Marks

Marks

Question 2

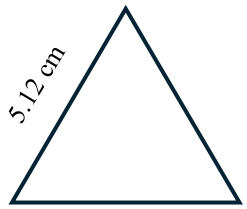
a) The population of the United States was 229.86 million in the year 1980, 281.48 million in the year 2000, and 339.44 million in the year 2020.

i) Calculate the percentage change in the United States population between 1980 and 2000. Give your answer correct to 1 decimal place. **2**

ii) Calculate the percentage change in the United States population between 2000 and 2020. Give your answer correct to 1 decimal place. **2**

iii) State, with a reason, the period in which the United States population grew slower, in percentage terms: between 1980 and 2000 or between 2000 and 2020. **1**

b) The sides of an equilateral triangle are each 5.12 cm long, rounded to 2 decimal places, as shown below:



i) Find the upper and lower bounds of the side length. **2**

ii) Find the upper and lower bounds of the perimeter, P, of the triangle. Give your answer to 2 decimal places. **2**

iii) Emma says that the height of the triangle is 4.43 cm. Say if Emma is correct, showing your working. **2**

c) Write 1230 as a product of prime factors. **2**

Marks

d) Consider the following expression:

$$12 + 2x(x - 1) + 4(x - 3)$$

i) Factorise the given expression. **2**

ii) Use your factorisation in (i) to solve the following equation: **3**

$$12 + 2x(x - 1) + 4(x - 3) = 4$$

e) Rearrange the following formula to make s the subject: **2**

$$6r - 4t = \frac{3}{s}$$

Total 20 Marks

Marks

Question 3

a) Consider the points $P = (-2, 3)$ and $Q = (6, -3)$.

i) Find the gradient of the straight line joining the points P and Q. **2**

ii) Write the equation of the straight line joining points P and Q in the form $y = mx + c$. **2**

b) The line l_1 has equation $3y - 6x - 6 = 0$

The line l_2 has equation $y = mx + 12$

i) Given that l_1 and l_2 are perpendicular, find the value of m. **2**

ii) Find the point P where lines l_1 and l_2 intersect. **4**

c) Consider the TWO (2) functions:

$f(x) = 4x - 3$

And

$g(x) = \frac{x}{2} + 8$

i) Find $f(f(x))$. **2**

ii) Find $g(f(3))$. **2**

iii) Find $f^{-1}(x)$. **2**

iv) Show that both $f(x)$ and $g(x)$ are neither odd nor even functions. **2**

v) Consider the function: **2**

$$h(x) = \frac{1}{4} f(x)$$

Explain the effect of this transformation on the graph of function $f(x)$.

Total 20 Marks

Marks

Question 4

a) Consider the function $y = x^3 - 6x^2 + 6x + 2$.

i) Calculate the value of y for the following SEVEN (7) x-values: **3**

$x = -1, 0, 1, 2, 3, 4, 5.$

ii) Plot this function and clearly indicate the SEVEN (7) points calculated in (i). Use the graph paper provided. **4**

iii) State the name of this type of graph. **1**

iv) Use the graph to solve the following equation: **3**

$$y = x^3 - 6x^2 + 6x + 2 = 0$$

v) Comment on the nature of any turning points on the graph. **2**

b) Consider the following quadratic equation:

$$2x^2 + 6x - 3 = 0$$

i) Cody says that this equation will have THREE (3) solutions. Without solving, briefly say why Cody is wrong. **1**

ii) Use the Quadratic Formula to solve the quadratic equation. Simplify your answer and leave it in surd form. **3**

c) Solve the inequality $11 < 9x^2 - 5$ **3**

Total 20 Marks

Marks

Question 5

a) A local gym recorded the ages of its members who attended a special fitness class for the past month. The ages of FORTY ONE (41) attendees are listed below:

23, 31, 45, 38, 29, 52, 41, 35, 27, 48,
33, 25, 50, 39, 44, 30, 26, 47, 36, 28,
51, 42, 34, 29, 46, 32, 24, 49, 37, 28,
53, 40, 35, 27, 43, 31, 25, 50, 36, 32, 41.

i) Say whether the data is continuous or discrete. Briefly explain your answer. **2**

ii) Present the data as an ordered stem and leaf diagram. **4**

ii) Find the median of the data. **2**

b) A group of 10 students were asked to record the average number of hours they slept per night in the week leading up to a major exam, and their score on that exam (out of 100). The results are shown in the table below:

Average hours of sleep per night	Exam score (out of 100)
6.5	72
8	90
5	60
7.5	85
7	78
6.25	74
5.5	65
8	92
6.8	75
7.2	84

i) State whether the data collected is quantitative or qualitative. Briefly explain your answer. **2**

ii) Construct a scatter graph to show this information. You should plot the average hours of sleep on the horizontal axis and the exam score on the vertical axis. Remember to label your axes appropriately. Use the graph paper provided. **5**

Marks

iii) Describe the correlation shown in the scatter graph. Briefly say how this correlation relates sleep with exam score. **3**

iv) Another student later says that they slept an average of 7 hours per night and received an exam score of 60. **2**

Say whether you think this is a relatively worse or better exam score than expected. Give ONE (1) reason for your answer.

Total 20 Marks

End of paper

Mathematics for University Study
Formulae sheet

Table of Content

1. Transformations of Functions

Vertical shift	$g(x) = f(x) + k$ (up for $k > 0$)
Horizontal shift	$g(x) = f(x - h)$ (right for $h > 0$)
Vertical reflection	$g(x) = -f(x)$
Horizontal reflection	$g(x) = f(-x)$
Vertical stretch	$g(x) = af(x)$ ($a > 0$)
Vertical compression	$g(x) = af(x)(0 < a < 1)$
Horizontal stretch	$g(x) = f(bx)$ $(0 < b < 1)$
Horizontal compression	$g(x) = f(bx)$ ($b > 1$)

2. The Quadratic Formula and Discriminants

Written in standard form, $ax^2 + bx + c = 0$, any quadratic equation can be solved using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, and c are real numbers and $a \neq 0$

The discriminant is defined as b^2-4ac.

- If $b^2 - 4ac = 0$, there is one rational solution
- If $b^2 - 4ac > 0$ and a perfect square, there are two rational solutions
- If $b^2 - 4ac > 0$ and not a perfect square, there are two irrational solutions
- If $b^2 - 4ac < 0$, there are two complex solutions

3. Properties of Inequalities

Addition Property	If $a < b$, then $a + c < b + c$
Multiplication Property	If $a < b$ and $c > 0$, then $ac < bc$ If $a < b$ and $c < 0$, then $ac > bc$

These properties also apply to $a \leq b$, $a > b$, and $a \geq b$

4. Logarithms

The **product rule for logarithms** can be used to simplify a logarithm of a product by rewriting it as a sum of individual logarithms.
$\log_b(MN) = \log_b M + \log_b N$ for $b > 0$

The **quotient rule for logarithms** can be used to simplify a logarithm of a quotient by rewriting it as the difference of individual logarithms.
$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

The **power rule for logarithms** can be used to simplify a logarithm of a power by rewriting it as the product of the exponent times the logarithm of the base.
$\log_b(M^n) = n\log_b M$

5. Sequences

The nth term of an arithmetic sequence is given by the explicit formula:
$a_n = a_1 + (n-1)d$

The nth term of a geometric sequence is given by the explicit formula:
$a_n = a_1 r^{n-1}$

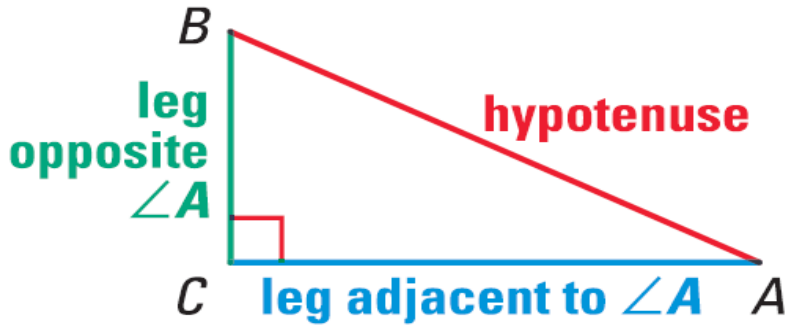
The formula for the **partial sum** of an arithmetic series is
$S_n = \frac{n(a_1 + a_n)}{2}$

The formula for the **partial sum** of a geometric series is
$S_n = \frac{a_1(1-r^n)}{1-r}$, $r \neq 1$

6. Geometry and Trigonometry

The sine and cosine ratios

- $\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

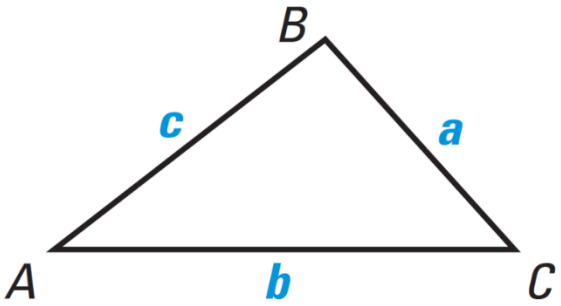


Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



Equation of a Circle
$(x-a)^2+(y-b)^2=r^2$

where (*a*, *b*) is the center and *r* is the radius

Perimeters and Areas for Shapes

Square
Side *s*
P = 4*s*
A = s^2

Rectangle
Length ℓ
Width *w*
P = 2ℓ + 2*w*
A = ℓw

Triangle
sides a, b, c
base b, height h
P = a + b + c
A = ½ *bh*

Circle
diameter d
radius r
C = 2πr
A = πr^2

Area of a Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

Where h is the height and b_1 and b_2 are the bases.

Area of a Rhombus

$$A = \frac{1}{2}d_1d_2$$

Where d_1 and d_2 are the diagonals.

Area of a Kite

$$A = \frac{1}{2}d_1d_2$$

Where d_1 and d_2 are the diagonals.

Area of a Regular Polygon

$$A = \frac{1}{2}Pa$$

Where P is the perimeter and a is the apothem

Surface Areas for Shapes

Surface Area of a Right Prism

$S = 2B + Ph$
Where S = surface area, B = base area,
P = perimeter of base, h = height of prism

Surface Area of a Right Cylinder

$S = 2\pi r^2 + 2\pi rh$
Where S = surface area, r = radius of base,
h = height of prism

Surface Area of a Regular Pyramid

$S = B + \frac{1}{2}P\ell$
Where B = base area, P = base perimeter, ℓ = slant height

Surface Area of a Right Cone

$S = \pi r^2 + \pi r\ell$
Where r = base radius, ℓ = slant height

Volumes for Shapes

Volume of a Prism

$V = Bh$
Where B = base area, h = height of prism

Volume of a Cylinder

$V = \pi r^2 h$
Where r = radius, h = height of cylinder

Volume of a Pyramid

$V = \frac{1}{3}Bh$
Where B = base area, h = height of pyramid

Volume of a Cone

$V = \frac{1}{3}\pi r^2 h$
Where r = radius, h = height of cone

Surface area of a Sphere

$S = 4\pi r^2$
Where r = radius

Volume of a Sphere

$V = \frac{4}{3}\pi r^3$
Where r = radius

Volume of the Frustum of a Pyramid

$V = \frac{1}{3}h(A_B + A_b + \sqrt{A_B A_b})$

where V = volume
h = height
A_B = area of the larger base
A_b = area of the smaller base

Volume of the Frustum of a Right Circular Cone

$$V = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

where V = volume
h = height
R = radius of larger base
r = radius of smaller base

7. Statistics

Mean

Ungrouped data:

$$\bar{x} = \frac{\sum x}{n}$$

Ungrouped frequency table:

$$\bar{x} = \frac{\sum fx}{n} = \frac{\sum fx}{\sum f}$$

Grouped frequency table:

$$\bar{x} = \frac{\sum fm}{n} = \frac{\sum fm}{\sum f}$$

Mode

Grouped data

Estimated mode = $L + \frac{(F_m - F_b) \times c}{(2F_m - F_b) - F_n}$

Median

Grouped data

Estimated median = $L_{med} + W_{med} \times \frac{\frac{n}{2} - C_{med}}{F_{med}}$

Quartiles

Ungrouped data:

$$LQ = \frac{1}{4}(n + 1)$$

$$UQ = \frac{3}{4}(n + 1)$$

Grouped data

Quartile = $L + \frac{(Q-P) \times I}{F}$

Mean deviation

Ungrouped data
Mean deviation $= \frac{\sum|x-\bar{x}|}{n}$

Grouped data
Mean deviation $= \frac{\sum f|x-\bar{x}|}{\sum f}$

Variance

Ungrouped data:
$$s^2 = \frac{\sum(x - \bar{x})^2}{n} = \frac{\sum x^2}{n} - \bar{x}^2$$

Grouped data:
$$s^2 = \frac{\sum f(x - \bar{x})^2}{\Sigma f} = \frac{\sum f x^2}{\Sigma f} - \left[\frac{\sum f x}{\Sigma f}\right]^2$$

Standard deviation

Ungrouped data:
$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$$

Grouped data:
$$s = \sqrt{\frac{\sum f x^2}{\Sigma f} - \left[\frac{\sum f x}{\Sigma f}\right]^2}$$

Coefficient of Variation
$$CV = \frac{s}{\bar{x}}$$

8. Kinematics
$$\boldsymbol{v = u + at}$$
$$\boldsymbol{v^2 = u^2 + 2as}$$
$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$