Mathematics for University Study

[Day] [Month] [Year]

Examination Paper

Sample Assessment

Answer ALL questions.

Clearly cross out surplus answers.

Time: 1.5 hours

The maximum mark for this paper is 100.

Any reference material brought into the examination room must be handed to the invigilator before the start of the examination.

Candidates are allowed to use a scientific calculator during this examination.

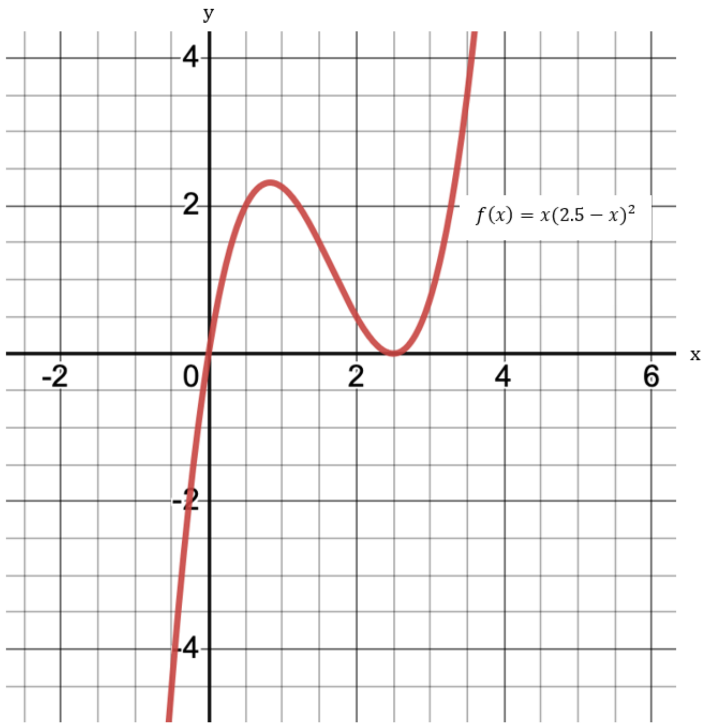
Answer ALL questions		
		Marks
Question 1		
a)	Simplify $(4\sqrt{5})^2$	**1**
	Mark Scheme $(4\sqrt{5})^2 = 4^2 \times (\sqrt{5})^2 = 16 \times 5 = 80$ ***(1 mark)***	
b)	Simplify $\frac{\sqrt{6}}{2\sqrt{6}+3\sqrt{2}}$, giving your answer in the form of $a + \sqrt{b}$, where a and b are integers.	**4**
	Mark Scheme $\frac{\sqrt{6}}{2\sqrt{6}+3\sqrt{2}} = \frac{\sqrt{6}}{2\sqrt{6}+3\sqrt{2}} \times \frac{2\sqrt{6}-3\sqrt{2}}{2\sqrt{6}-3\sqrt{2}}$ ***(1 mark)*** $= \frac{12-3\sqrt{12}}{24-18}$ ***(1 mark)*** $= \frac{12-6\sqrt{3}}{6}$ ***(1 mark)*** $= 2 - \sqrt{3}$ ***(1 mark)***	
c)	Let $f(x) = 3x^3 - 19x^2 + 33x - 9$ Prove that $(x - 3)$ is a factor of $f(x)$	**2**
	Mark Scheme $f(3) = 3(3)^3 - 19(3)^2 + 33(3) - 9$ $f(3) = 81 - 171 + 99 - 9 = 0$ ***(1 mark)*** Therefore, $(x - 3)$ is a factor of $f(x)$ ***(1 mark)***	

d)	Let $f(x) = 3x^3 - 19x^2 + 33x - 9$ Using algebra, show that $f(x) = 0$ has only two distinct roots.	**4**
	Mark Scheme $3x^3 - 19x^2 + 33x - 9 \div (x - 3) = 3x^2 - 10x + 3$ (By long division) ***(1 mark)*** $3x^2 - 10x + 3 = (3x - 1)(x - 3)$ (Any factorisation method can be used) ***(1 mark)*** Therefore, $3x^3 - 19x^2 + 33x - 9 \div (x - 3) = (3x - 1)(x - 3)^2$ ***(1 mark)*** The TWO (2) distinct roots are $x = 3, \frac{1}{3}$ ***(1 mark)***	
e)	Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin, find the TWO (2) possible values of k.	**2**
	Mark Scheme $f(x + k) = 0$ $x + k = 3, \frac{1}{3}$ ***(1 mark)*** $x = 0$ *at the origin*. Therefore, $k = 3, \frac{1}{3}$ ***(1 mark)***	
f)	The line l_1 has equation $3x + 2y - 5 = 0$ The line l_2 has equation $y = mx + 5$, where m is a constant. Given that l_1 and l_2 are perpendicular, find the value of m to THREE (3) decimal places.	**2**
	Mark Scheme $Line\ l1: y = -1.5x + 2.5$ $Line\ l2: y = mx + 5$ $Lines\ l1\ and\ l2\ are\ perpendicular\ to\ each\ other: -1.5 \times m = -1$ ***(1 mark)*** Therefore, $m = 0.667\ (i.e., \frac{2}{3})$ ***(1 mark)***	

g)	Lines l_1 and l_2 in part (f) above meet at the point P. Find the x and y co-ordinates of P.		**5**
	Mark Scheme $\frac{2}{3}x + 5 = -\frac{3}{2}x + \frac{5}{2}$ ***(1 mark)*** $\frac{2}{3}x + \frac{3}{2}x = -5 + \frac{5}{2}$ ***(1 mark)*** $\frac{13}{6}x = -\frac{5}{2}$ ***(1 mark)*** $x = -\frac{30}{26} = -\frac{15}{13}$ ***(1 mark)*** $y = -\frac{3}{2}(-\frac{15}{13}) + \frac{5}{2} = \frac{55}{13}$ ***(1 mark)***		
			Total 20 Marks
Question 2			
a)	$g(x) = \frac{3x+4}{x-2}$ $\quad x \geq 4$		
	i)	Find $g(g(4))$	**2**
		Mark Scheme $g(4) = \frac{(3\times4)+4}{4-2} = 8$ ***(1 mark)*** $g(g(4)) = \frac{(3\times8)+4}{8-2} = \frac{14}{3}$ ***(1 mark)***	
	ii)	State the range of g	**1**
		Mark Scheme $3 < g \leq 8$	

	iii)	Find $g^{-1}(x)$	**3**
		Mark Scheme $g = \frac{3x+4}{x-2}$ ***(1 mark)*** $gx - 2g = 3x + 4$ $x\,(g - 3) = 4 + 2g$ ***(1 mark)*** $x = \frac{(4 + 2g)}{(g - 3)}$ $g^{-1}(x) = \frac{(4+2x)}{(x - 3)}$ ***(1 mark)***	

Marks

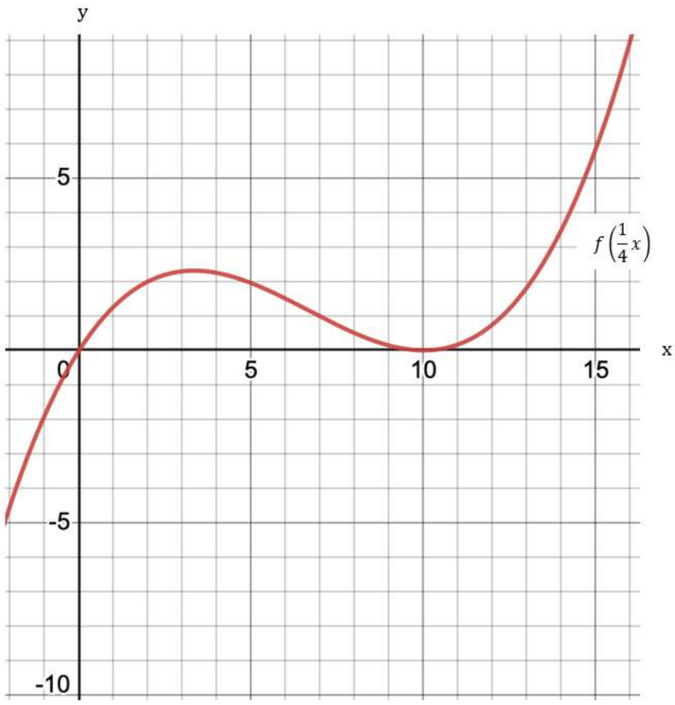
b)	The figure below shows a sketch of the curve with equation $y = f(x)$ where $f(x) = x(2.5 - x)^2$ $\quad\quad x \in \mathcal{R}$ The curve passes through the origin and touches the x-axis at point (2.5, 0). There is a maximum point at (0.83, 2.32) and a minimum point at (2.5, 0). On separate diagrams, sketch the curve with equation: (a) $y = f(\frac{1}{4}x)$ (b) $y = f(x + 4)$ On each sketch, indicate clearly the co-ordinates of: • any points where the curve crosses or touches the x-axis, • the pint where the curve crosses the y-axis, • any maximum or minimum point.  	**8**

Marks

Mark Scheme

(a) $y = f(\frac{1}{4}x)$***:***

Transformation will produce a horizontal stretch (see list of mathematical formulae pamphlet) – will still cross at the origin; the y-co-ordinates of the minimum and maximum points will not change, but x-coordinates will be 4 times bigger for this type of transformation. (2 marks)

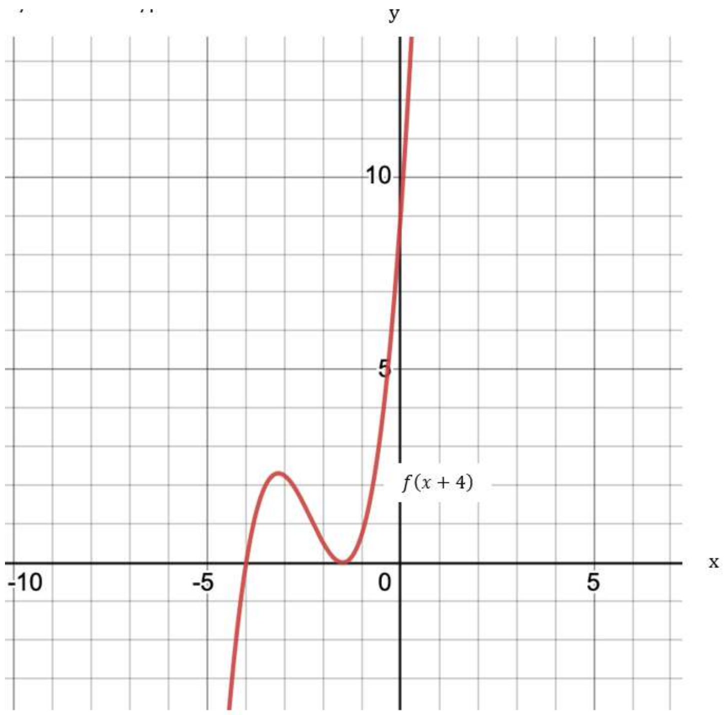


- ***Crosses the origin***
- ***Touches x-axis at (10, 0) – also a minimum point (1 mark)***
- ***Maximum point at (3.33, 2.32) (1 mark)***

(b) $y = f(x + 4)$***:***

Transformation will produce a horizontal shift (see list of mathematical formulae pamphlet) – will cross at -4 now because of the shift; the y-co-ordinates of the minimum and maximum points will not change, but x-coordinates will shift to the left by 4 for this type of transformation. (2 marks)

Marks



- ***Crosses the x-axis at (-4, 0)***
- ***Touches x-axis at (-1.5, 0) – also a minimum point*** ***(1 mark)***

- ***Maximum point at (-3.17, 2.32)***
- ***Crosses the y-axis at (0, 9)*** ***(1 mark)***

			Marks
c)	Find the set of values of x for which:		
	i)	$2 - 3x < 7 + 5x$	**2**
		Mark Scheme $2 - 3x < 7 + 5x$ $-5 < 8x$ ***(1 mark)*** $x > -\frac{5}{8}$ ***(1 mark)***	
	ii)	$5x^2 + 13x - 6 < 0$	**2**
		Mark Scheme $5x^2 + 13x - 6 < 0$ $(5x - 2)(x + 3) < 0$ [Factorise] ***(1 mark)*** $-3 < x < \frac{2}{5}$ ***(1 mark)***	
	iii)	both $2 - 3x < 7 + 5x$ and $5x^2 + 13x - 6 < 0$	**2**
		Mark Scheme $x > -\frac{5}{8}$ ***(1 mark)*** and $x < \frac{2}{5}$ ***(1 mark)*** [Between -3 and $-\frac{5}{8}$ will not satisfy the first inequality]	
			Total 20 Marks

Marks

Question 3		
a)	Solve the equation $x\sqrt{2} - \sqrt{32} = x$ Writing the answer as a surd in simplest form.	**3**
	Mark Scheme $x\sqrt{2} - \sqrt{32} = x$ $x\sqrt{2} - 4\sqrt{2} = x$ ***(1 mark)*** $x(\sqrt{2} - 1)(\sqrt{2} + 1) = 4\sqrt{2} \times (\sqrt{2} + 1)$ ***(1 mark)*** $x = (8 + 4\sqrt{2})$ ***(1 mark)***	
b)	Solve the equation $$9^{(4x-3)} = \frac{1}{3\sqrt{3}}$$	**3**
	Mark Scheme $9^{(4x-3)} = \frac{1}{3\sqrt{3}}$ $(3^2)^{(4x-3)} = (3^{-1})(3^{-\frac{1}{2}})$ ***(1 mark)*** $3^{(8x-6)} = 3^{-\frac{3}{2}}$ $8x - 6 = -\frac{3}{2}$ $8x = \frac{9}{2}$ ***(1 mark)*** $x = \frac{9}{16}$ ***(1 mark)***	
c)	Let $g(x) = 2x^3 + 3x^2 - 29x - 60$ Use the factor theorem to show that $g(x)$ is divisible by $(x - 4)$	**2**
	Mark Scheme $g(3) = 2(4)^3 + 3(4)^2 - 29(4) - 60$ $g(3) = 128 + 48 - 116 - 60 = 0$ ***(1 mark)*** Therefore, $(x - 4)$ is a factor of $g(x)$ ***(1 mark)***	

		Marks
d)	For $g(x)$ in part (c) above, showing all your working, write $g(x)$ as a product of THREE (3) linear factors.	**4**
	Mark Scheme $2x^3 + 3x^2 - 29x - 60 \div (x - 4) = 2x^2 + 11x + 15$ (By long division) ***(1 mark)*** $2x^2 + 11x + 15 = (2x + 5)(x + 3)$ (Any factorisation method can be used) ***(1 mark)*** Therefore, $2x^3 + 3x^2 - 29x - 60 \div (x - 4) = (2x + 5)(x + 3)(x - 4)$ ***(1 mark)*** The three roots are $x = 4, -\frac{5}{2}, -3$ ***(1 mark)***	
e)	Using algebra, find all solutions of the equation $3x^3 + 14x^2 - 5x = 0$	**3**
	Mark Scheme $3x^3 + 14x^2 - 5x = 0$ $x(3x^2 + 14x - 5) = 0$ ***(1 mark)*** $x(3x - 1)(x + 5) = 0$ ***(1 mark)*** $x = 0, \frac{1}{3}, -5$ ***(1 mark)***	
f)	find all real solutions of: $3(y-3)^6 + 14(y-3)^4 - 5(y-3)^2 = 0$	**3**
	Mark Scheme $(y-3)^2 = 0 \Longrightarrow y = 3$ ***(1 mark)*** $(y-3)^2 = \frac{1}{3} \Longrightarrow y = 3 \pm \sqrt{1/3}$ ***(1 mark)*** $(y-3)^2 = -5 \Longrightarrow no\ real\ solution$ ***(1 mark)***	

g)	Given $\frac{16^{(x+2)}}{4^{(y-3)}} = 64$, express y in terms of x, writing your answer in simplest form.	**2**
	Mark Scheme $\frac{16^{(x+2)}}{4^{(y-3)}} = 64$ $\frac{(4^2)^{(x+2)}}{4^{(y-3)}} = 4^3$ $4^{(x+2-y+3)} = 4^3$ ***(1 mark)*** $x - y + 5 = 3$ $y = x + 2$ ***(1 mark)***	

Total 20 Marks

Question 4

a)	The line l passes through the points A (4, 6) and B (2, 9).		
	i)	Find an equation for l in the form $ax + by + c = 0$ where a, b and c are integers.	**4**
		Mark Scheme $y = mx + c$ $m = \frac{(9-6)}{(2-4)} = \frac{-3}{2}$ ***(1 mark)*** $6 = \frac{-3}{2}(4) + c \Longrightarrow c = 12$ ***(1 mark)*** $y = \frac{-3}{2}x + 12 \Longrightarrow 2y = -3x + 24$ ***(1 mark)*** $3x + 2y - 24 = 0$ ***(1 mark)***	
	ii)	Find the length AB, leaving your answer in surd form.	**2**
		Mark Scheme $Length\ AB = \sqrt{(9-6)^2 + (2-4)^2}$ $(\mathbf{1\ mark})$ $= \sqrt{9+4} = \sqrt{13}$ ***(1 mark)***	

	iii)	The point C has coordinates (p, 6) and $AC = CB$. Find the value of p.	**3**

Mark Scheme

$AC = \sqrt{(p-4)^2}$ ***(1 mark)***

$CB = \sqrt{(2-p)^2 + 3^2}$ ***(1 mark)***

$(p-4)^2 = (2-p)^2 + 9 \Rightarrow 4p = 3 \Rightarrow p = \frac{3}{4}$ ***(1 mark)***

b)	Use proof by exhaustion to show that for $n \in \mathbb{N}, n \leq 4$, $(n+1)^4 > 4^n$	**3**

Mark Scheme

(2 marks for a correctly filled out table as below. Deduct 1 mark for each error up to a maximum of 2 marks.)

n	$(n+1)^4$	4^n	Comment
1	16	4	16 > 4
2	81	16	81 > 16
3	256	64	256 > 64
4	625	256	625 > 256

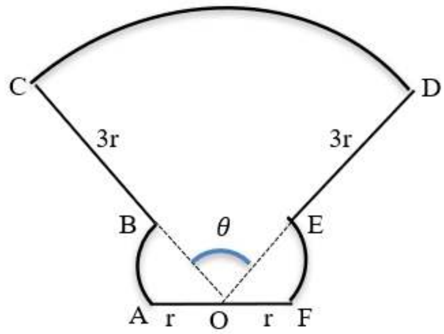
So, if $n \leq 4, n \in \mathbb{N}, then\ (n+1)^4 > 4^n$ ***(1 mark)***

c) The shape OABCDEFO shown below is a design for a logo.

In the design:

- OAB is a sector of a circle centre O and radius r
- Sector OFE is congruent to sector OAB
- ODC is a sector of a circle centre O and radius 3r
- AOF is a straight line

Given that the size of angle COD is θ radians,



i) Show that the area of the logo is $\frac{1}{2}r^2\,(8\theta + \pi)$ **4**

Mark Scheme

$Area$=[$\theta 2\pi\pi(3r)2+\pi - \theta 4\pi\pi r2 Area = [\frac{\theta}{2\pi}(\pi)(3r)^2 + \frac{(\pi-\theta)}{4\pi}(\pi r^2)(2)]$ ***(1 mark)***

$Area = [\frac{9r^2\theta}{2} + \frac{(\pi-\theta)r^2}{2}]$ ***(1 mark)***

$Area = \frac{1}{2}r^2[9\theta + (\pi - \theta)]$ ***(1 mark)***

$Area = \frac{1}{2}r^2[8\theta + \pi]$ ***(1 mark)***

ii) Find the perimeter of the logo, giving your answer in the simplest form in terms of r, θ and π **4**

Mark Scheme

$Perimeter = [\frac{\theta}{2\pi}(2\pi)(3r) + \frac{(\pi-\theta)}{4\pi}(2\pi r)(2)] + 6r$ ***(1 mark)***

$Area = [3r\theta + (\pi - \theta)r + 6r]$ ***(1 mark)***

$Area = r[3\theta + (\pi - \theta) + 6]$ ***(1 mark)***

$Area = r[6 + 2\theta + \pi]$ ***(1 mark)***

Total 20 Marks

Marks

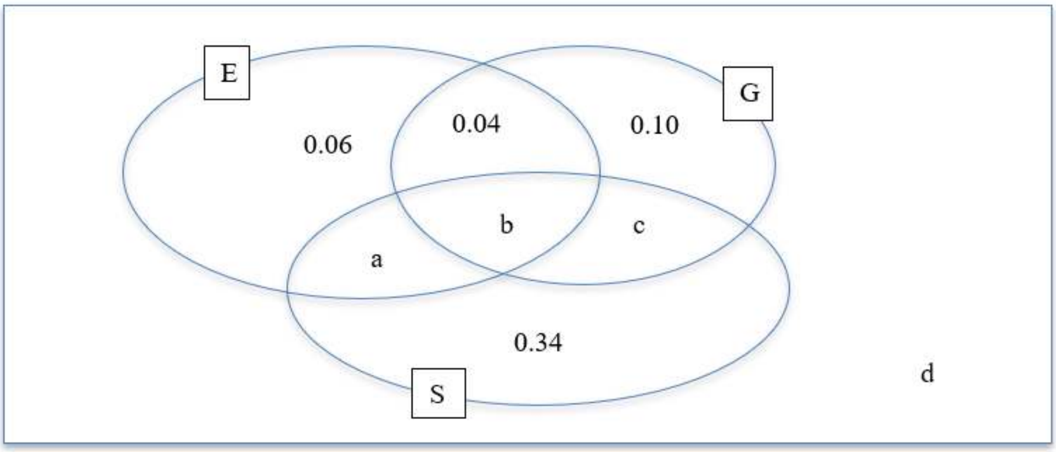
Question 5

a) A university produces THREE (3) newsletters. ONE (1) newsletter is about the environment, ONE (1) is about games technology and ONE (1) is about sports.

A student at the university is selected at random and the events E, G and S are defined as follows:

- E is the event that the student reads the newsletter about the environment.
- G is the event that the student reads the newsletter about games technology.
- E is the event that the student reads the newsletter about sports.

The Venn diagram below, where a, b, c and d are probabilities, gives the probability for each subset.



i) Find the proportion of students in the university who read exactly one of these newsletters. **1**

Mark Scheme

$0.06 + 0.10 + 0.34 = 0.50$ ***(1 mark)***

ii) If no student read all the t newsletters and P(E) = 0.24, find: **3**

(i) the value of b

(ii) the value of a

Mark Scheme

$(i)\ P(E \cap G \cap S) = b = 0$

$(ii)\ P(E) = 0.24$

$0.06 + 0.04 + a + b = 0.24$

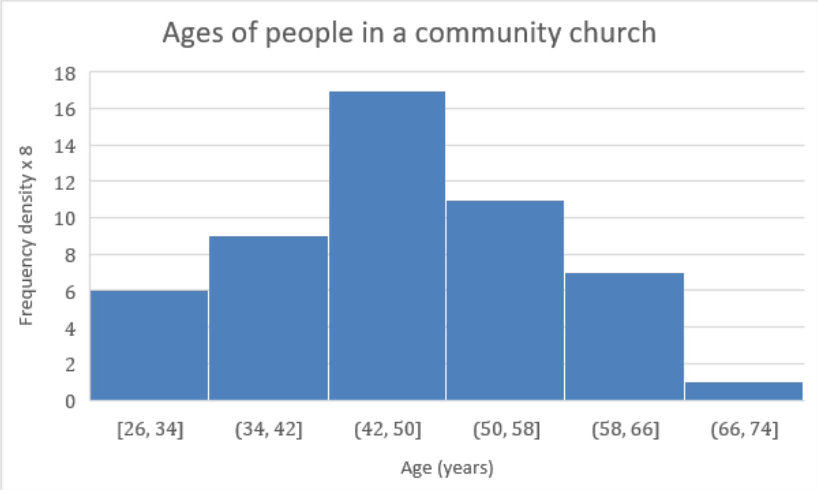
$0.10 + a = 0.24 \Rightarrow a = 0.14$

(1 mark for working out, 1 mark for correct value of a, 1 mark for correct value of b)

			Marks
	iii)	Given that P (S|G) =$\frac{6}{11}$ Find (i) the value of c (ii) the value of d	**4**
		Mark Scheme (i) P (S|G) =$\frac{(c+b)}{(c+b+0.10+0.04)} = \frac{6}{11} \Longrightarrow \frac{c}{(c+0.14)} = \frac{6}{11} \Longrightarrow c = 0.168$ ***(1 mark for working out, 1 mark for correct value of c)*** $(ii)\ 0.06 + 0.10 + 0.34 + 0.14 + 0.168 + d = 1 \Longrightarrow d = 0.192$ ***1 mark for working out, 1 mark for correct value of d)***	

b)	The histogram below summarises the ages of 51 people in a community church.		

Figure — Ages of 51 people in a community church



	i)	Draw the ogive for this frequency distribution.	**4**

Mark Scheme

Upper boundary	Frequency (f)	Cumulative frequency
34	6	6
42	9	15
50	17	32
58	11	43
66	7	50
74	1	51

(2 marks for correctly filled out table, 2 marks for a correctly drawn ogive)

ii) estimate the median age **1**

Mark Scheme

At cumulative frequency of 26 (50 percentile), median age = 47.2 years ***(1 mark)*** (see also the ogive diagram).

iii) Estimate the mean age **3**

Mark Scheme

Midpoint (X)	Frequency (f)	(X)(f)	
30	6	180	
38	9	342	
46	17	782	
54	11	594	
62	7	434	
70	1	70	
Sum	51	2402	
	Mean =	47.10	years

(2 marks for correctly filled out table, 1 mark for the correct mean figure)

c) A university made a profit of £100,000 in its first year of operation, Year 1.

A model is developed for future university operations that the yearly profit will increase by 12% each year so that the yearly profits will form a geometric sequence.

According to the model,

i) Show that the profit in Year 3 will be £140493. **1**

Mark Scheme

$Year\ 4 = £100000(1.12^3) = £140493$ ***(1 mark)***

ii) Find the first year when the yearly profit will exceed £1 million. **2**

Mark Scheme

$100000(1.12^{n-1}) > 1000000$ ***(1 mark)***

$(1.12^{n-1}) > 10$

$n - 1 > \frac{log10}{log1.12} \Rightarrow n - 1 > 20.31 \Rightarrow n > 21.31\ [22nd\ year]$ ***(1 mark)***

	iii)	Find the total profit for the first 20 years of trading, giving your answer to the nearest £1000	**1**
		Mark Scheme $S_{20} = \frac{a(1-r^n)}{(1-r)} = \frac{100000(1-1.12^{20})}{(1-1.12)} = £7.205\ million$ ***(1 mark)***	
			Total 20 Marks

End of paper

Learning Outcomes matrix

Question	Learning Outcomes / Assessment Criteria assessed	Marker can differentiate between varying levels of achievement
1	LO1, LO3, LO4	Yes
2	LO3, LO4	Yes
3	LO3, LO4	Yes
4	LO2, LO3, LO4	Yes
5	LO2, LO5, LO6	Yes

Grade descriptors

<table>
<tr><th>Learning Outcome</th><th>Pass
(40-59%)</th><th>Merit
(60-69%)</th><th>Distinction
(70-100%)</th></tr>
<tr><td>1.Be able to develop fundamental knowledge, skills and understanding of number and algebra and be able to perform and solve a range of algebraic calculations, equations and inequalities</td><td rowspan="6">Demonstrates an adequate awareness and understanding of concepts, terminology, principles and processes with a reasonable application and use of mathematical principles, tools and techniques and satisfactory reference to theory through an adequate ability to perform calculations correctly using appropriate techniques and methods.</td><td rowspan="6">Demonstrates a consistent and accurate awareness and understanding of concepts, terminology, principles and processes with a detailed application and use of mathematical principles, tools and techniques and precise reference to theory through a robust ability to perform calculations consistently correctly using appropriate techniques and methods.</td><td rowspan="6">Demonstrates an outstanding awareness and understanding of concepts, terminology, principles and processes with a highly comprehensive and sophisticated application and use of mathematical principles, tools and techniques and critical and meticulous reference to theory through an excellent ability to perform calculations consistently correctly and to the highest standard using appropriate techniques and methods.</td></tr>
<tr><td>2.Acquire, select and apply mathematical techniques to solve sequence, ratio, proportion, rates of change and geometry problems</td></tr>
<tr><td>3.Be able to mathematically, solve, reason, make deductions and inferences and draw conclusions within mathematics and other contexts</td></tr>
<tr><td>4.Comprehend, interpret and communicate mathematical information in a variety of forms appropriate to the information and context.</td></tr>
<tr><td>5.Recognise and apply the fundamentals of statistics and be able to present data in graphical form</td></tr>
<tr><td>6.Recognise and apply the fundamentals of probability</td></tr>
</table>